An Inverse Approach to MR-EPT Reconstruction

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Purpose: We present a novel approach to imaging an organ's electrical properties with MRI using a variant of the recently developed MR Electrical Properties Tomography (MR-EPT). MR-EPT makes certain simplifications to the Maxwell Equations describing the propagation of the B1 RF field; in particular, the electrical conductivity is shown to be proportional to the Laplacian of the B1 field phase (Eq. 1). Computing the Laplacian (second derivatives) of the phase is extremely sensitive to noise and undesirable. A recent approach using Gauss' theorem to express conductivity as a surface integral of the gradients of B1 phase, reduces this computation to a first order derivative and makes image reconstruction less sensitive to noise. However, even in this case noisy MR data can limit reconstruction quality. Here, we propose an inverse formulation, in which the spatial distribution of conductivity is taken as a parameter to be fitted; specifically, a simulated phase is computed from an initial estimate of the conductivity distribution and fitted to the MR measured B1 phase. This has the advantage of not requiring computation of any phase derivatives. This approach is applicable to both MR-EPT conductivity and permittivity imaging, and has the potential of improving MR-EPT in in-vivo applications.

Methods: MR-EPT conductivity imaging is based on Eq. (1), where the conductivity, σ, is approximately proportional to the Laplacian of the phase of the transmit B1 field $\phi(H^+(r))$ at any point in space, r. Conversely, given a spatial distribution of σ, it is possible to express the resulting phase as Eq. (2), where a simulated phase ϕ_{sim} is computed from $\sigma(r)$ by solving the Poisson equation.

Eq. 1: $$\sigma(r) \approx \frac{1}{\omega\mu}\nabla^2\phi(H^+(r))$$ **Eq. 2:** $$\nabla^2(\phi_{sim}) = \omega\mu\sigma(r)$$

Reconstruction of $\sigma(r)$ is therefore based on: 1) taking an initial estimate of $\sigma(r)$, for example a uniform distribution, 2) computing the corresponding simulated phase ϕ_{sim} by solving Eq. (2), and 3) updating $\sigma(r)$ based on the discrepancy between the measured and simulated phases $\| \phi(H^+) - \phi_{sim} \|^2$, in the least squares sense. This approach is known as an inverse formulation, and the update of σ can be expressed using Eq. (3), where J is the Jacobian of the conductivity to phase mapping expressed by Eq. 2, L is a regularization matrix, and α is a regularization parameter used to stabilize the inversion. The initial or current conductivity distribution is iteratively updated as $\sigma_{new} = \sigma_{cur} + \delta\sigma$, where σ_{cur} is the current estimate of $\sigma(r)$. Since the problem linear, a single iteration of Eq. 3 results in the sought conductivity.

Eq. 3: $$\delta\sigma = (J^TJ + \alpha L^TL)^{-1}(J^T(\phi(H^+(r)) - \phi_{sim} + \alpha L^TL\sigma))$$

Results: A phantom (Fig. 1a) was created by placing gelatin slices of increasing thicknesses (5, 10, 15, and 20mm) in a saline-filled container. The conductivity contrast between gelatin and saline was approximately 2:1 (NaCl was added to gelatin resulting in a conductivity of ~1.8 S/m). $CuSO_4$ (an MR contrast agent) was added to the gelatin**,** but not to the saline. An MR amplitude image was acquired on a Philips Achieva 3T platform, with a standard 3D SE sequence (Fig. 1b). Phase data from the 3D SE was recorded and utilized for conductivity image reconstruction. The second spatial derivatives of the phase images were computed with Savitzky-Golay filters using the central point and three adjacent points on each side of the central pixel. Conductivity is reconstructed with Eq. 1 and later smoothed with a Gaussian filter with a standard deviation of two pixels to reduce noise. The resulting image is shown in Fig. 2a; the less conductive gelatin appears darker than the saline. Fig. 2b shows the estimated conductivty distribution reconstructed using the **proposed** inverse formulation. This approach does not exhibit the significant boundary artifacts and lower resolution of Fig. 2a, in which the dependance on derivati**v**e computation is sensitive to the image boundary and **stable derivative estimation** requires information from several neighbouring pixels, thus reducing resolution and smoothing sharp changes. Fig. 3 illustrates the fitting process; the green line is the initial guess (uniform conductivity of 0.1 S/m), red is the measured phase, and blue is the fitted phase.

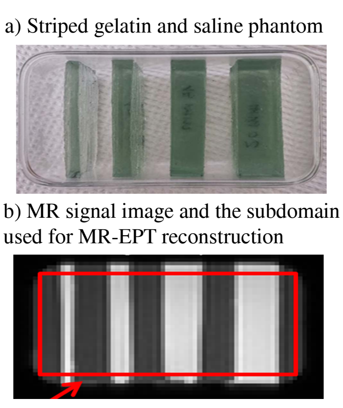

Figure 1 - Phantom (a) and its MR magnitude image (b).

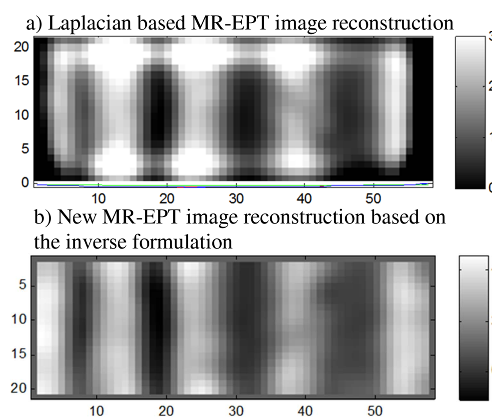

Figure 2 - MR-EPT Reconstructions: Laplacian based (a) and, inverse formulation based (b).

Discussion/Conclusion: The inverse formulation presented here enables conductivity reconstruction by matching the MR-recorded phase information. The primary advantage of this approach is that is does not require differentiation of the phase data. Some smoothing is enforced by a regularization functional, but the strength of this functional can be controlled (e.g. functionals allowing sharp contrast changes, such as Total Variation, can be employed). Another approach, using prior structural information from MRI, can be used to build ad-hoc regularization functionals that permit sharper transitions at particular locations. In summary, we believe the approach proposed and demonstrated here establishes a new technique for reconstructing MR-EPT images which can leverage recent developments in inverse problems and ultimatley produce more clinically useful electrical property images.

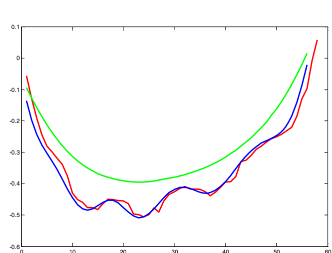

Figure 3 – Phase fitting process**.**