Accuracies of the Laplacian based phase processing methods

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Introduction

The Laplacian of phase can be obtained directly from the original phase images without phase unwrapping[1,2]. This has been frequently used in background phase removal in quantitative susceptibility mapping[3,4]. Currently there are mainly two ways to calculate the Laplacian of phase. One method is to use the wrapped phase differences and discrete Laplacian operator[1], denoted as "**discrete Laplacian operators**"; the other method is to use trigonometric functions and Fourier transform to calculate the Laplacian and its inverse[2–4], denoted as "**continuous Laplacian operators**". Former studies have found errors associated with veins in the phase images processed using continuous Laplacian operators[3,4]. In this study, both discrete and continuous Laplacian operators were evaluated using simulated and *in vivo* data, in order to understand the source of the error associated with veins in the processed phase images.

Theory and Method

The total phase information can be written as $\varphi = \varphi_B + \varphi_L$. It is known that the unwrapped background phase φ_B satisfies the Laplace's equation[2] and thus $\Delta\varphi = \Delta\varphi_L$, where Δ represents the Laplacian operator, which can be calculated as: $\Delta\varphi = \varphi(i+1,j,k) + \varphi(i,j+1,k) + \varphi(i,j,k+1) + \varphi(i-1,j,k) + \varphi(i,j-1,k) + \varphi(i,j,k-1) - 6\varphi(i,j,k)$. This can be calculated using the wrapped phase differences[1] using the original phase image with wraps φ_w as: $\arctan(\exp((-1)^{0.5}(\varphi_w(i+1) - \varphi_w(i))))$. The processed phase can be calculated as $\varphi_L = FT^{-1}\{FT[M\Delta\varphi]L^{-1}(k)\}$, where $L^{-1}(k)$ is a regularized inverse of the Laplacian operator in Fourier domain (with a truncation threshold of 0.015), M is an eroded binary mask[4]. The above calculations are denoted as "**discrete Laplacian operators**" in this study. Alternatively, it was shown that[2] $\Delta\varphi = \cos\varphi_w(\Delta\sin\varphi_w) - \sin\varphi_w(\Delta\cos\varphi_w)$. The Laplacian of any function *f* can be calculated as[3]: $\Delta f = -4\pi^2/N^2 FT^{-1}[k^2 FT(f)]$. The inverse Laplacian was calculated similarly. The above calculations are denoted as "**continuous Laplacian operator**" in this study. These two ways of phase processing were evaluated using simulated and *in vivo* data. The phase images of the 3D brain model were generated using forward calculation[5] with B_0=3T, TE=10ms. The background phase was added by including the air-sinuses with a susceptibility of 9ppm. For the *in vivo* data, B_0=3T, TE=14.3ms, voxel size $0.5 \times 0.5 \times 0.5 mm^3$. For the simulated data, susceptibility maps were generated using truncated k-space division method[6] with a threshold 0.1. Susceptibility of the veins and relative errors were measured.

Results

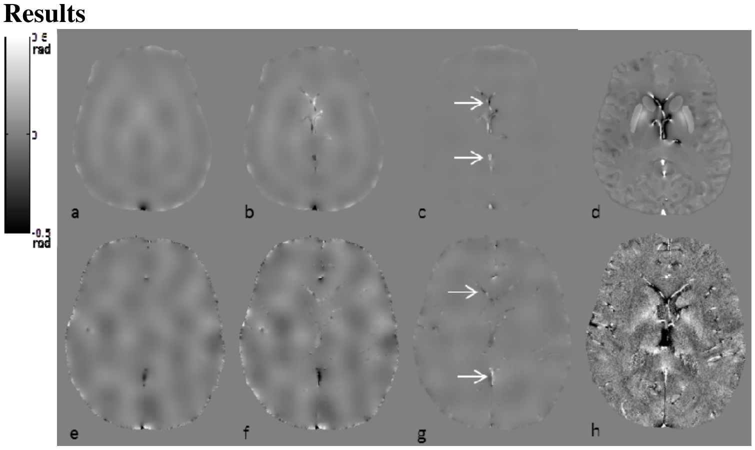

Fig. 1. **a** to **d**: simulated data results, **e to f:** *in vivo* data results**.** **(a)** Error in the processed phase image induced by discrete Laplacian operators. **(b)** Error induced by continuous Laplacian operators. **(c) a-b.** **(d)** The original simulated phase image. **(e)** Error induced by discrete Laplacian operators. **(f)** Error induced by continuous Laplacian operators. **(g) e-f.** **(h)** Reference phase image processed by 3D phase unwrapping + SHARP (kernel size 6pixels). The white arrows indicate that the discrete and continuous Laplacian operators lead to different results for the veins. This difference is likely to be caused by errors in the continuous Laplacian operators, as shown in the error maps **b** and **f.**

The errors in the processed phase images induced by the discrete and continuous operators are shown in **Fig.1**. The simulated data results show that continuous Laplacian operators lead to significant errors associated with veins. This was also seen from the *in vivo* data results, as shown in **Fig. 1.f**. For the brain model, susceptibility values of the veins were measured as 0.41±0.03 ppm (relative error 9%) and 0.36±0.04ppm (relative error 20%), using the discrete and continuous Laplacian operators, respectively.

Discussions and Conclusions

In recent studies, the continuous Laplacian operators were usually used, and errors/differences were observed to be associated with the veins. This study demonstrated that phase images processed using the continuous operator may lead to an additional 11% error in the estimated susceptibility. This is mainly attributed to the use of the continuous Laplacian operator, since it is implicitly assumed that the function is continuous and differentiable. This assumption is violated on edges of the veins, where the field is discontinuous. On the other hand, the discrete Laplacian operators based on wrapped phase difference do not have such limitations. Instead, it is assumed that the absolute phase difference between two neighboring pixels is less than π, which is satisfied for most tissues with low susceptibility values at low TEs. However, in order to avoid any noise amplification, it might be better to unwrap the phase first by calculating the integer number of multiples of 2π, than using the Laplacian directly in the later processing steps.

Reference 1.Ghiglia and Pritt. *Two-dimensional phase unwrapping: theory, algorithms, and software*. (Wiley,1998).2.Schofield&Zhu. *Opt. Lett.* **28,** 1194–1196 (2003).3.Li et al. *NeuroImage* **55,** 1645–1656 (2011).4.Schweser et al. MRM **69,** 1582–1594 (2013). 5. Salomir et al. Concept Magn. Reson. B, 19B(1) 26–34 (2003). 6.Haacke et al. JMRI **32,** 663–676 (2010).