Quantitative signal analysis in the dipole field of a single vessel

Christian H Ziener[1]

[1]*Radiology, German Cancer Research Center, Heidelberg, Baden-Württemberg, Germany*

Target audience This work targets investigators with an interest in quantitative susceptibility weighted imaging and diffusion weighted imaging with special emphasis on theoretical aspects of MR imaging and signal formation.

Purpose Susceptibility weighted imaging is of paramount interest to visualize vessels and to quantify parameters of capillary networks [1,2]. Also the BOLD effect in functional imaging relies on the susceptibility difference of blood and the surrounding parenchyma [3]. The knowledge of the local magnetization around a blood filled vessel is crucial to determine the susceptibility inside the vessel which finally allows quantifying the susceptibility effects for SWI or oxygenation levels for fMRI. The first full analytical solution of the local magnetization for dephasing and diffusion in a dipole field around a single vessel is given by solving the Bloch-Torrey equation without approximations.

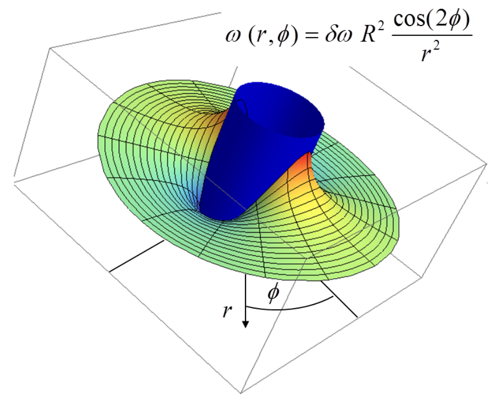

Fig.1: Single blood filled vessel and the surrounding dipole field. The susceptibility effects are summarized in the frequency shift $\delta\omega$ and r and ϕ are polar coordinates.

Methods A single blood filled vessel with radius R is considered. Due to the susceptibility difference a local dipole field $\omega(r,\phi)$ around the vessel is generated (see Fig. 1) in which the dephasing of nuclear spins occurs. The local magnetization is also influenced by diffusion around the vessel. The magnetization is governed by the Bloch-Torrey equation:

$$\partial_t m(r,\phi,t) = [D\Delta + i\omega(r,\phi)]m(r,\phi,t)$$

where Δ is the Laplacian and D is the diffusion coefficient of the surrounding parenchyma [4].

Results The local magnetization can be calculated with the following equation:

$$m(r,\phi,t) \approx 4m_0 e^{-\frac{t}{T_2}} \frac{\sqrt{\pi D t}}{r} e^{-\frac{r^2}{8Dt}} \sum_{m=0}^{\infty} \frac{A_0^{(2m)} ce_{2m}(\phi, i\delta\omega\, R^2/[2D])}{2^{k_m}\Gamma((1+k_m)/2)} M_{-\frac{1}{2},\frac{k_m}{2}}\left(\frac{r^2}{4Dt}\right)$$

where *ce* are the Mathieu functions and *M* denotes the Whittaker function [5]. The eigenvalue k_m is visualized for different values of the parameter $\delta\omega R^2/D$ in Fig. 2. For small values of the parameter $\delta\omega R^2/D$ all eigenvalues are purely real, which leads to a decay of the magnetization. For larger values of the parameter the eigenvalues become complex and oscillating components of the magnetization have to be taken into account.

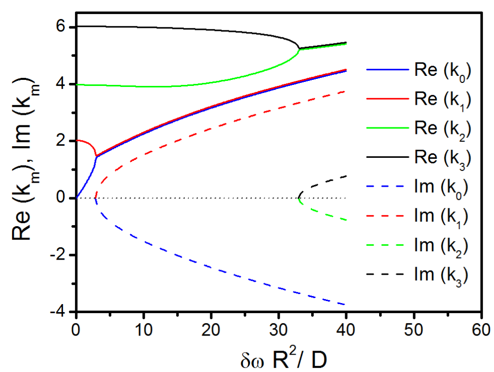

Fig.2: Eigenvalues in dependence on the tissue parameters.

The local magnetization around a single vessel with the radius R=5µm, diffusion coefficient D=1µm^2/ms and frequency shift $\delta\omega$=200/s is visualized in Fig. 3 for t=10 ms after the excitation pulse. In the vicinity of the vessel the magnetization exhibits the same symmetry as the dipole field.

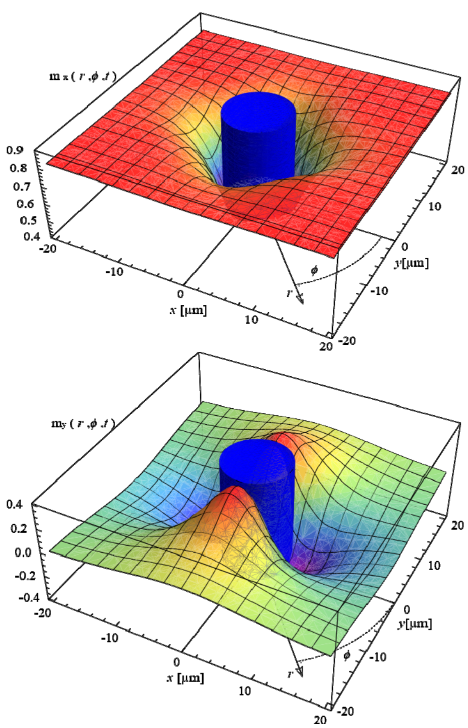

Fig.3: Components of the transverse magnetization obtained from the analytical solution.

Discussion & Conclusion A full analytic solution of the Bloch-Torrey equation for dephasing in the dipole field around a single vessel is presented. In contrast to previous work in field of spin dephasing theory no approximations were made. The analysis also includes the diffusion effects, which are dominating in the case of small vessels where the static dephasing conditions are not fulfilled. With these results at hand, it is possible to determine the local magnetization and, thus, susceptibility weighted images can be interpreted correctly.

References 1. Reichenbach JR, Neuroimage 2012;62:2083-2100, 2. Sedlacik J, MRM 2007;58:1035-1044, 3. Haacke EM, Neuroimage 2012;62:923-929, 4. Torrey HC, Phys Rev 1956;104:563-565, 5. Ziener CH, *et al.*, Phys Rev E 2012;85:051908