

Field perturbations due to hollow spheres with anisotropic magnetic susceptibility

Matthew Cronin¹, Samuel Wharton¹, Penny Gowland¹, and Richard Bowtell¹

¹Sir Peter Mansfield Magnetic Resonance Centre, The University of Nottingham, Nottingham, Notts, United Kingdom

Introduction: The magnetic field perturbations produced when simple structures composed of materials with isotropic magnetic susceptibility are exposed to a uniform magnetic field have been well documented [1], but the effect of material of anisotropic magnetic susceptibility has been less fully explored. Here, we show that interesting and potentially useful effects can be generated by using shaped structures composed of anisotropic material. We focus on spherical shells composed of anisotropic material characterised by a cylindrically-symmetric susceptibility tensor whose principal axis is radially-oriented. It has previously been reported that this arrangement occurs in spherical lipoproteins, as a result of the arrangement of lipid chains in the outer shell, and that it generates a uniform frequency offset in the lipoprotein core whose magnitude depends on the shell structure [2]. This counterintuitive behaviour was investigated in detail in the work described here, by applying field mapping sequences at 3 T to a phantom containing spherical shells composed of pyrolytic graphite sheet (PGS), a material with highly anisotropic magnetic properties [3]. Measurements of the field perturbations produced by shells of varying size were used to test the relationship between radius and internal field offset and to highlight the potential use of these structures for generating tuneable contrast in MRI.

Theory: The susceptibility of an anisotropic material can be represented in the local frame using a susceptibility tensor of the form shown in Eq. 1, where χ_I and χ_A ($\ll 1$) characterise the isotropic and anisotropic components of the susceptibility. If a spherical shell of inner/outer radius, r_i/r_o , composed of this material with the

$$\underline{\chi} = \chi_I \underline{I} + \begin{bmatrix} -\chi_A/2 & 0 & 0 \\ 0 & -\chi_A/2 & 0 \\ 0 & 0 & \chi_A \end{bmatrix} \quad \text{Eq. [1]}$$

principal axis of the tensor oriented in the radial direction is exposed to a uniform magnetic field, $\mathbf{B} = B_0 \mathbf{k}$, the induced magnetization is given by Eq. 2.

$$\mu_0 \mathbf{M} = H_0 \left(\frac{3\chi_A}{4} \sin 2\theta (\cos \phi \mathbf{i} + \sin \phi \mathbf{j}) + \left(\chi_I + \frac{\chi_A}{2} (3 \cos^2 \theta - 1) \right) \mathbf{k} \right) \quad \text{Eq. [2]}$$

Then, by evaluation of the magnetic scalar potential produced by this magnetization distribution, it can be shown that the magnetic field perturbation, ΔB , inside and outside the sphere is given by Eq. 3. This shows that there is a uniform field offset inside the sphere which depends only on χ_A and a standard dipole field outside the sphere, which depends only on χ_I . If the shell is thin, so that $r_o - r_i = t$ ($\ll r_o, r_i$), the internal field offset is given by $\chi_A B_0 t / r_i$.

$$\begin{aligned} \Delta B &= \chi_A B_0 \ln \left(\frac{r_o}{r_i} \right) & [r < r_i] & \quad \text{Eq. [3]} \\ \Delta B &= \chi_I B_0 \left(\frac{r_o^3 - r_i^3}{r^3} \right) (\cos^2 \theta - \frac{1}{3}) & [r > r_o] & \end{aligned}$$

Methods: Five, thin-walled plastic spheres with radii of 5, 10, 12.5, 19, and 25 mm were covered with a 25 μm layer of PGS (Panasonic EYGS121803). PGS is strongly diamagnetic and magnetically anisotropic, with a cylindrically symmetric tensor whose principal component is normal to the sheet. The spheres were filled with water and set in an 18 cm diameter spherical agar phantom. The phantom was scanned at 3 T using a dual gradient echo B_0 -mapping sequence ($TE_1/TE_2/TR = 4.2/5.7/20$ ms) with 1.5 mm isotropic voxels. Field maps (in Hz) were exported for further processing in Matlab.

Results: Figure 1 presents an axial cross section through a region of the frequency map spanning the centres of the five spheres. It shows that a uniform, negative frequency/field offset is produced inside each sphere, with a magnitude that increases as the radius decreases. The field outside the spheres is relatively homogeneous, indicating that any external dipolar field is relatively weak. Figure 2 shows that the mean internal frequency is proportional to the inverse of the sphere radius, as predicted by theory. The line of best fit ($R^2 = 0.99$) has a slope of -690 ± 40 Hz mm. With $\gamma B_0 / 2\pi = 128$ MHz and $t = 25 \mu\text{m}$, this yields a value of χ_A of -216 ± 13 ppm, which is in agreement with previously reported values of the anisotropy of PGS [3].

Discussion: The results demonstrate that spherical shells composed of anisotropic material characterised by a susceptibility tensor whose principal component is radially-oriented produce a spatially uniform, internal field/frequency offset. This finding is consistent with a previous spectroscopy-based study of (~ 10 nm radius) spherical lipoproteins [2]. Figure 2 shows that, in agreement with theory, the internal frequency offset scales with the inverse radius of the spherical shell for fixed thickness, highlighting the fact that the offset can be tuned by simple variation of shell geometry. Since the dipolar field generated outside the shell is independent of the susceptibility anisotropy and depends only on the isotropic susceptibility, an internal frequency shift can potentially be generated without producing significant external field perturbation. This behaviour is evident in Fig. 1 from the minimal frequency perturbation outside the spheres and is a consequence of the small value of χ_I compared with χ_A in PGS. These findings indicate that miniaturised hollow spheres composed of anisotropic material could potentially form the basis of useful tuneable contrast agents [4], since they could provide a geometrically controlled internal frequency offset, with minimal perturbation of the signal from outside the spheres.

References: [1] Chu *et al.* 1990 MRM 13:239. [2] Lounila *et al.* 1994. PRL 72:4049. [3] Wilson *et al.* MRM 49: 906. [4] Zabow *et al.* 2008 Nature 453, 1058.

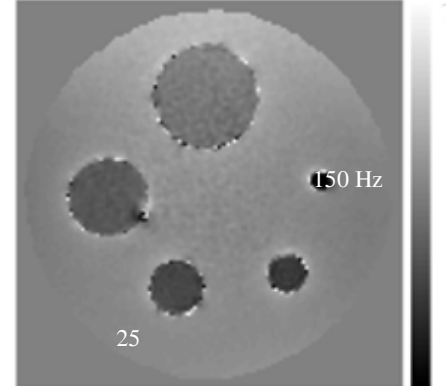


Figure 1 – frequency map showing PGS-wrapped spheres with radii of 5, 10, 12.5, 19 and 25 mm.

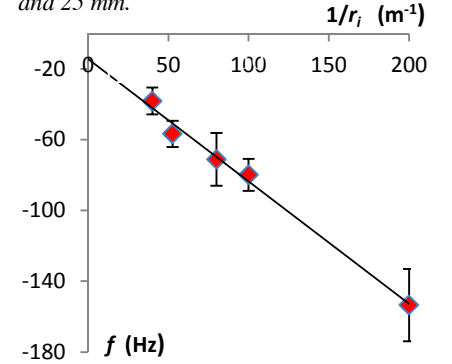


Figure 2 – internal frequency shift as a function of $(r_i)^{-1}$. Error bars show the standard deviation over voxels.