Direct & accelerated parameter mapping using the unscented Kalman filter

Li Zhao[1] and Craig H. Meyer[1,2]

[1]*Biomedical Engineering, University of Virginia, Charlottesville, Virginia, United States,* [2]*Radiology, University of Virginia, Charlottesville, Virginia, United States*

Purpose: Tissue parameter mapping shows substantial promise for improved disease characterization. For example, T1 and T2 changes correlate with pathology in tumors, stroke, and cardiomyopathy. However, long acquisition times have slowed the adoption of parameter mapping. Several measurements are required along the parameter encoding direction to compute an accurate parameter map; the classic example is imaging at multiple TEs for T2 mapping. Accelerated parameter mapping methods based on compressed sensing have recently been introduced and can achieve acceleration rates of 4-8 [1,2]. These methods adopt the traditional paradigm of image formation followed by parameter estimation and use the underlying sparsity of the signal evolution to accelerate image acquisition. The goal of this study was to explore a new paradigm of directly estimating the parameter map from the k-space data itself, thus combining image reconstruction and optimal parameter estimation. The resulting method yields high acceleration rates, improved accuracy, and fast parameter map reconstruction.

Methods: In this study, we applied the unscented Kalman filter (UKF) to accelerate T2 mapping. The unscented Kalman filter is an optimal method for estimating the state of a nonlinear system from noisy measurements [3]. The central idea of our method is that the parameter, T2, is the state of the system, and our task is to estimate that state at each pixel. In MRI, we do not directly observe the parameter, but rather measure signals that are nonlinear functions of the parameter. We model this measurement process and use the UKF to recursively improve our estimate of the system state. The resulting parameter estimate is optimal in a least-squares sense when all of the data has been processed.

The UKF describes the evolution of the system state and the measurement of the system using the following recursive relationships:

$$T2_{k+1} = f(T2_k, w_k) = T2_k \qquad\qquad z_{k+1} = h(T2_k, v_k) = F_k \rho e^{-\frac{t_k}{T2_k}} + v$$

f is the state transition function (unity in this case), h is the measurement function, and z is the acquired data. F_k is a Fourier transform with an under-sampling pattern defined for each time t_k. v is measurement noise, assumed to be white Gaussian noise. ρ is the proton density. We studied two different versions of the UKF. In the single UKF (S-UKF), we assumed that the proton density is known via a separate measurement and estimate T2 as shown here. In the double UKF (D-UKF), both T2 and proton density are included in the system state and estimated. The UKF proceeds through a series of time update and measurement update equations as described in [3].

Simulated and experimental data acquisition was performed using a 2D multiple-contrast Cartesian spin echo pulse sequence. Localization along the readout direction was performed using a 1D FFT prior to the UKF [4]. Data was retrospectively under-sampled in ky-parameter space by factors of 2-8 from the origin fully sampled. For quantitative evaluation of parameter estimation accuracy, an analytical phantom [5] was used to simulate the T2 mapping acquisition, reconstruction and parameter estimation process. 4 ROIs were selected with T2 values of 50, 80, 120, 250 ms, covering the range of values expected in gray and white matter. Experimental volunteer data was collected using a Siemens 3T Trio scanner with a 12-channel head coil with the following parameters: TR 2.5 s, slice thickness 5 mm, FOV 220 × 220 mm and matrix size 192×192. 70 spin echoes were acquired with echo spacing of 5.5 ms. T2 maps calculated using S-UKF, D-UKF and compressed sensing with K-SVD [2] were quantified by comparison to fully sampled data using normalized RMS error (NRMSE), structural similarity index (SSI), and SNR. Computation time was recorded for each method. Data processing and image reconstruction were performed using MATLAB 2012b.

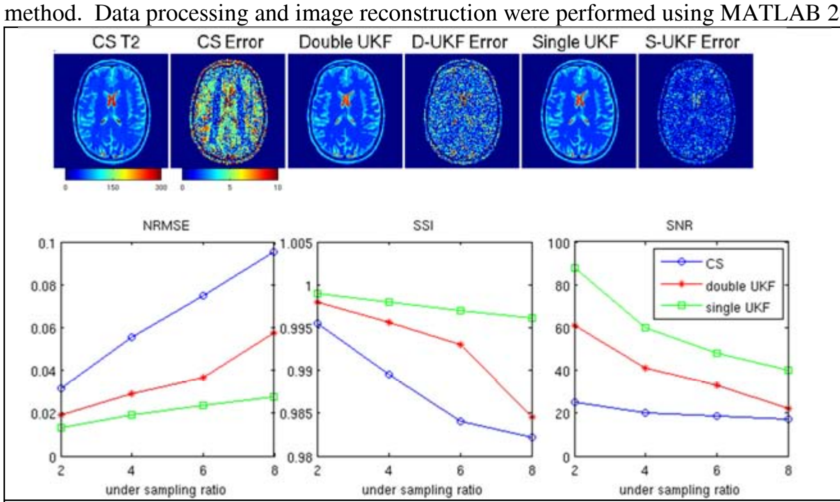

Figure 1. Numerical phantom simulation. Top: T2 maps and resulting error with under-sampling rate 8 for the three methods. Bottom: NMRSI, SSE and SNR for under-sampling rates 2-8. With T2 and proton density estimation, D-UKF yields lower T2 map estimation error and better similarity and SNR than CS with K-SVD. Given prior knowledge of proton density, S-UKF improves the T2 map further.

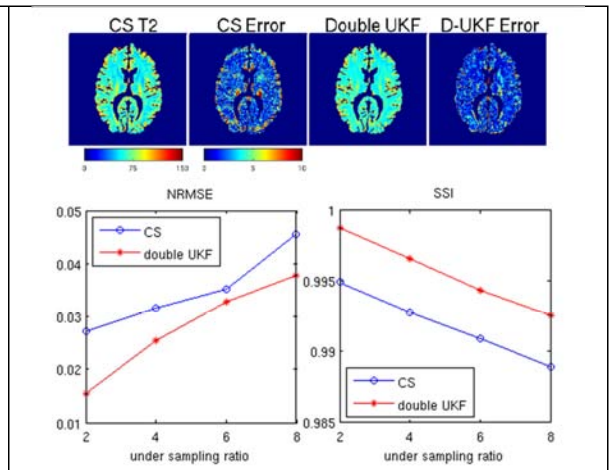

Figure 2. Experimental T2 parameter mapping. Top: T2 maps and resulting error with under-sampling rate 8. Bottom: NMRSE and SSI for under-sampling rates 2-8. D-UKF yielded more accurate T2 maps with higher structural similarity than CS with K-SVD.

Results: The results are summarized in Figs. 1 and 2 and in the Table. The D-UKF yielded more accurate T2 maps than CS at all under-sampling rates for both simulated and experimental data. With known proton density, the S-UKF improves the estimation performance further. The computation time was shorter for D-UKF and S-UKF than CS.

Computation time

	CS	D-UKF	S-UKF
Single iteration (s)	11.5	1.05	0.26
Number of iteration	10-50*	100	100
Map calculation (s)	48.5	0	0
Total time (s)	163.5-623.5	105	26

*Doneva et al. [2] suggested 6-35 iterations

Discussion: By combining image reconstruction and parameter estimation, the unscented Kalman filter enables 8X-accelerated parameter mapping with excellent accuracy and short computation time. The method requires no regularization parameters. UKF methods can easily be adapted to estimate other parameters, including T1.

Reference: 1. Block et al. IEEE Trans Med Imaging, 2009;28:1759-1769. 2. Doneva et al. MRM 2010;64:1114-1120. 3. Wan et al. IEEE AS-SPCC, 2000;153-158. 4. Feng et al. MRM 2012;69:1346-1356. 5. Guerquin-Kern et al. IEEE Trans Med Imaging. 2012;31:626-636.