Accurate T2 Mapping with Sparisty and Linear Predictability Filtering

Xi Peng[1], Leslie Ying[2], Xin Liu[1], and Dong Liang[1]

[1]*Paul C. Lauterbur Research Centre for Biomedical Imaging, Shenzhen Key Laboratory for MRI, Shenzhen Institutes of Advanced Technology, Shenzhen, Guangdong, China,* [2]*Department of Biomedical Engineering, Department of Electrical Engineering, The State University of New York at Buffalo, Buffalo, New York, United States*

INTRODUCTION: Accelerating the acquisition process of T2 mapping via sparse sampling has drawn considerable attention recently [1-6]. Prior knowledge such as image sparisty [7] and spatiotemporal partial separability [8] has been exploited widely so as to suppress the artifacts associated with undersampling. However, due to non-ideal conditions in practical settings (i.e., insufficient sparsity/rank and coherent sampling), system errors occur in the T2-weighted image series and the subsequent relaxation map especially with high reduction factors and noisy measurements. In this work, we address this problem by integrating the prior information (i.e., exponential functions) on the temporal signals into the image reconstruction step. This is in contrast to the conventional wisdom where the image reconstruction and parameter mapping are performed independently. Specifically, we add linear predictability [9] as a nonlinear filter to constrain the exponential behavior of the temporal signal which may be disturbed during sparse reconstruction. The proposed method was evaluated on an in-vivo brain dataset and shows promising results.

THEORY AND METHOD: The signal function $\rho(r,t_m)$ of a T2-weighted image series generated by a Carr-Purcell-Meiboom-Gill(CPMG) spin echo can be written as: $\rho(r,t_m)=\rho_0(r)\sum_{l=1}^{L}\exp\left[-m\Delta\mathrm{TE}/T_{2,l}(r)+i\theta(r)\right]$ (1), where $\rho_0(r)$ and $T_{2,l}(r)$ represent the proton density and T2 map respectively. $\theta(r)$ is the phase term which is assumed to be the same for all the images. The r and $t_m=m\Delta\mathrm{TE}$ denote spatial coordinate and the m-th echo time, $\Delta\mathrm{TE}$ is the echo spacing, L denotes the number of linearly combined exponential terms, which is application dependent. According to Eq. (1), $\rho(r,t_m)$ is linearly predictable: $\rho(r,t_m)=\sum_{l=1}^{L}\alpha_l\rho(r,t_{m-l})$ and a Hankel matrix can be formed for each spatial location:

$$H[\rho]=\begin{bmatrix}\rho(t_1) & \rho(t_2) & \cdots & \rho(t_K)\\ \rho(t_2) & \rho(t_3) & \cdots & \rho(t_{K+1})\\ \vdots & \vdots & \vdots & \vdots\\ \rho(t_{M-K+1}) & \rho(t_{M-K+2}) & \cdots & \rho(t_M)\end{bmatrix}$$

Actually, the Hankel matrix will have rank L as long as $\rho(r,t_m)$ has form (1). Thus the low rankness is an intrinsic property associated with the signal model and can be taken into account for reconstructing the image series from undersampled data:

$$\hat{\boldsymbol{\rho}}=\underset{rank(H[\boldsymbol{\rho}])=L}{\arg\min}\ \|\mathbf{F}_u\boldsymbol{\rho}-\mathbf{d}\|_2^2+\lambda\left|\boldsymbol{\Psi}_s\boldsymbol{\rho}\boldsymbol{\Theta}_{pca}\right|_1 \qquad (2)$$

where $\boldsymbol{\rho}\in\mathbb{C}^{N\times M}$ and $\mathbf{d}\in\mathbb{C}^{U\times M}$ are the matrix form of image functions and measurements. N is the total number of image pixels and M is the number of images. $\mathbf{F}_u\in\mathbb{C}^{U\times N}$ denotes the undersampled Fourier encoding. $\boldsymbol{\Psi}_s$ is a spatial sparsifying transform (i.e., wavelet) and $\boldsymbol{\Theta}_{pca}\in\mathbb{C}^{M\times M}$ is the principle component matrix learned from the low resolution data whose column capture the major variation along the temporal dimension [4]. The regularization term $\left|\boldsymbol{\Psi}_s\boldsymbol{\rho}\boldsymbol{\Theta}_{pca}\right|_1$ enforces temporal redundancy and spatial sparsity simultaneously. We propose to solve problem (2) in a nonlinear filtering framework [10] as summarized in Algorithm 1. Specifically, image sparsity and temporal redundancy are imposed through soft-thresholding in wavelet-PCA domain (**Step 2**). To promote linear predictability, we conduct low rank matrix approximation using singular value decomposition (SVD) on the Hankel matrix formed at each spatial location of the current image function (**Step 5**). Function Hankelize($\cdot$) restores the Hankel structure of the low rank approximated matrix by averaging along the anti-diagonals and replacing each element by the mean value of that anti-diagonal. Data consistency was enforced right after each nonlinear filtering (**Steps 4** and **6**).

Algorithm 1 CS-**LP** reconstruction

Input: $\mathbf{d}\in\mathbb{C}^{U\times M}$

Initialization: $\boldsymbol{\rho}^{(0)}=\mathbf{F}_u^H\mathbf{d}$, $k=0$

Iterate:

1. $\alpha^{(k)}=\boldsymbol{\Psi}_s\boldsymbol{\rho}^{(k)}\boldsymbol{\Theta}_{pca}$, % transfer to wavelet-pca domain

2. $\alpha^{(k+1)}=\mathrm{sign}\left(\alpha^{(k)}\right)\max\left(\left|\alpha^{(k)}\right|-\tau,0\right)$, % soft thresholding

3. $\boldsymbol{\rho}^{(k+1)}=\boldsymbol{\Psi}_s^H\alpha^{(k+1)}\boldsymbol{\Theta}_{pca}^{-1}$, % transfer to image domain

4. $\boldsymbol{\rho}^{(k+1)}=\mathbf{F}_c^H\mathbf{F}_c\boldsymbol{\rho}^{(k+1)}+\boldsymbol{\rho}^{(0)}$ with $\mathbf{F}_c=\mathbf{F}-\mathbf{F}_u$, % data consistency

5. for each spatial location

a). $[u\ \sigma\ v]=\mathrm{svd}\left(H\left[\rho^{(k+1)}\right]\right)$, $\hat{H}^{(k+1)}=\sum_{l=1}^{L}u_l\sigma_l v_l^H$

b). $\hat{H}^{(k+1)}=\mathrm{Hankelize}\left(\hat{H}^{(k+1)}\right)$

c). extract $\hat{\boldsymbol{\rho}}^{(k+1)}$ from $\hat{H}^{(k+1)}$

6. $\hat{\boldsymbol{\rho}}^{(k+1)}=\mathbf{F}_c^H\mathbf{F}_c\hat{\boldsymbol{\rho}}^{(k+1)}+\boldsymbol{\rho}^{(0)}$ %data consistency

Until convergence.

EXPERIMENT AND RESULT: We validated the proposed method using a fully sampled multi-contrast brain dataset acquired on 3T scanner (MAGNETOM Trio, SIEMENS, Germany) using a turbo spin echo sequence with a 12-channel head coil array (matrix size=192×192, FOV=192mm×192mm, slice thickness=3.0mm, ETL=15, ΔTE=8:8ms, TR=4000ms, bandwidth=362Hz/pixel). Informed content was obtained from the imaging subject in compliance with the Institutional Review Board (IRB) policy. Undersampled measurements were retrospectively obtained using variable density undersampling along the phase encoding dimension. Conventional CS based reconstruction with PCA as the sparsifying transform along the temporal direction was also conducted for comparison. Reconstruction errors of the T2-weighted images and the estimated T2 maps are presented in Fig.1 and Fig.2. As can be seen, the proposed method outperforms the standard CS-based method for all frames at a reduction factor of 4. The T2 maps estimated from the CS reconstruction suffered from obvious aliasing artifacts, while the proposed technique successfully suppressed the artifacts to an acceptable level with all reduction factors.

DISSCUSION AND CONCLUSION: This work proposed a novel method to reconstruct the T2-weighted images from undersampled k-space data by incorporating linear predictability to constrain the exponential behavior of the temporal signal which may be biased during standard sparse reconstruction. The proposed technique can also be applied to other parameter mapping applications where signal of multiple-exponential-decay is involved.

ACKNOWLEDGEMENT: We would like to acknowledge NSFC 61102043, 81120108012, 11301508, the Basic Research Program of Shenzhen JC201104220219A and the Shenzhen Peacock Plan KQCX20120816155710259.

REFERENCE: [1] M. Doneva, *MRM* 2010. pp. 1114-1120. [2] J.V. Velikina, *MRM* 2013. pp. 1263-1273. [3] F.H. Petzschner, *MRM* 2011. pp. 706-716. [4] C. Huang, *MRM* 2012. pp.1355-1366. [5]W. Li, *MRM* 2012. pp. 1127-1134. [6] B. Zhao, *ISMRM* 2012. pp. 2233. [7] Lustig M, *MRM* 2007. pp. 1182-1195. [8] Z.-P. Liang, *ISBI* 2007. pp. 988-991. [9] H.M. Nguyen, *IEEE TBME* 2013. pp. 78-89. [10] L.B. Montefusco, *IEEE TSP* 2009. pp. 2494-2502.

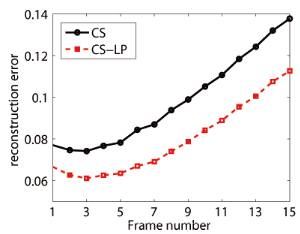

Fig. 1. The relative reconstruction error using the CS method and the proposed CS-LP method for all frames at R=4.

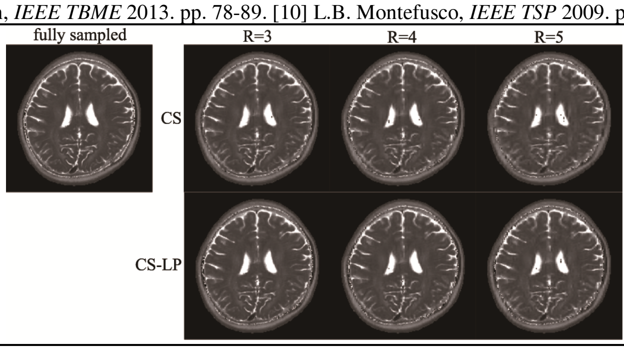

Fig. 2. Estimated T2 maps of the brain dataset using the reconstruction from the CS and proposed CS-LP method with reduction factor R of 3, 4 and 5.