Use of L1-norm solution to Impose Spatial Smoothness Constraints in Quantitative T2 Relaxometry

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Target Audience: Researchers interested in using prior information in T2 relaxometry quantification.

Introduction: Qunatitative T2 relaxometry (QT2R) has been successfully used to monitor tissue damages in various demyelinating neurological disease such as multiple sclerosis [1] and stroke, along with detecting abnormal tissues in muscle and liver and even cancerous tissues. However, because of the ill-posedness of the problem, the returned T2 distributions and subsequently the tissue fraction maps are very sensitive to signal to noise ratio (SNR), requiring high SNR of >500 for robust data fitting. Conventional L2-norm regularization [2] imposing the temporal smoothness of the T2 distribution improves the stability of the solution. But, improvement is not adequate at low SNR. Kumar et al [3] introduced spatial smoothness constraints using L2-norm.

Here, we compare the performance of an L1-norm solver against L2-norm solver.

Theory: Assuming the underlying T2 distribution consists of discrete T2 points logarithmically chosen over a range of relevant T2 values, the signal at any echo time TE_k for a single voxel is given by: $\mathbf{y} = \mathbf{Ax} + \boldsymbol{\varepsilon}$, with $\mathbf{A}_{ki} = \exp(-TE_k/T_2(i))$ with $\mathbf{y}$ the signal decay column form and $\mathbf{x}$ the column vector consisting of all volume fractions α_i for respective T2 values of T2(i), and $\boldsymbol{\varepsilon}$ denotes the noise vector (white Gaussian). The corresponding multiple voxels forward equation can be written as: $\bar{\mathbf{y}} = \mathbf{A}_{ex}\bar{\mathbf{x}} + \bar{\varepsilon}; \bar{\mathbf{x}} \geq 0$ where the single-voxel quantities $\mathbf{x}$, $\mathbf{y}$, $\boldsymbol{\varepsilon}$ are collected into multi-voxel column vectors $\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\varepsilon}$ and $\mathbf{A}_{ex}$ is the block diagonal matrix. To improve the stability of reconstruction with respect to noise, the prior expectations regarding the spatial smoothness of tissue organizations is introduced. The spatial smoothness using L2-norm [3] minimize $\hat{\mathbf{x}} = \arg\min_{\bar{\mathbf{x}}} \left\|\mathbf{A}_{ex}\bar{\mathbf{x}} - \bar{\mathbf{y}}\right\|_2^2 + M_T\left\|\bar{\mathbf{x}}\right\|_2^2 + \mu_S\left\|\mathbf{D}_S\bar{\mathbf{x}}\right\|^2; \mathbf{x} \geq 0$ (1), whereas the form using L1-norm minimize $\hat{\mathbf{x}} = \arg\min_{\bar{\mathbf{x}}} \left\|\mathbf{A}_{ex}\bar{\mathbf{x}} - \bar{\mathbf{y}}\right\|_2^2 + M_T\left\|\bar{\mathbf{x}}\right\|_2^2 + \mu_S\left\|\mathbf{D}_S\bar{\mathbf{x}}\right\|_1; \mathbf{x} \geq 0$ (2)

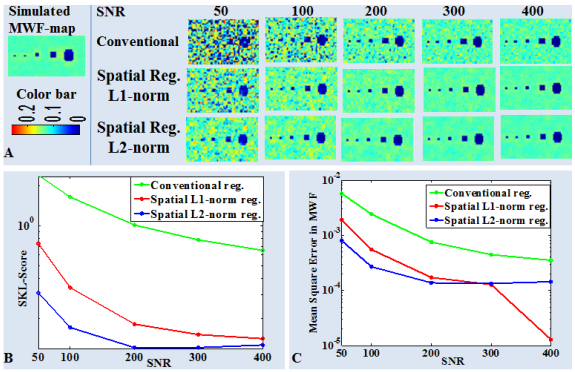

Fig.1. (A) Simulated area is compared against three different reconstruction approaches at five different SNRs. (B) Average symmetric Kullbeck-Leibler (SKL) score as a function of SNRs for three different approaches (separate averages for lesions and surrounding matrix reported). (C) Relative mean square of reconstructed MWF maps as a function of SNRs.

where M_T is the diagonal matrix with voxelwise temporal regularization μ_T along its diagonal and μ_S is spatial regularization parameter. The first term imposes the data fidelity, while the second term, the conventional regularization term, penalizes large values in inferred T2 distributions. The third term imposes spatial constraints. Matrix D_S is first difference operator, the norm $||D_S\,\bar{\mathbf{x}}||$ penalizes non-smooth solutions for each T2-points.

Data and Methods:

Simulation: A numerical phantom consisting of lesions (single pool with T2 of 100 ms) with varying sizes (1-8 voxels wide) surrounded by matrix (two pools with geometric means T2 of 30 ms and 100 ms) was used to evaluate the developed L1-solver against L2-solver.

Experiment: We acquired QT2R data at 7T (Bruker) using two mice undergoing transient middle cerebral artery occlusion, a stroke model. The QT2R data was acquired using 2D CPMG sequence with the following parameters: axial FOV = 20 mm, matrix size = 128x96, partial phase FOV = 0.75, receiver bandwidth = 454 kHz, # slices = 8, slice thickness = 0.5 mm, TR = 2000 ms, # echoes = 32, echo spacing = 5.4 ms; SNR ~200, only even echoes were further evaluated as these are not affected by B1-error for the CPMG sequence, acquisition time = 30 min.

Methods: The optimum values of temporal regularization constant μ_T, the diagonal elements of M_T in eqs.(1) and (2), are allowed to vary voxelwise and are chosen by L-curve method as described in [3]. The spatial regularization parameter μ_S is assumed to be spatially invariant and is chosen as 1000*median(μ_T-map). Eq (1) is minimized as described in [3]. Eq (2) is solved using split Bregman method [4] where L1-problem is approximated as small L2-subproblems. These small L2-subproblems are solved using modified version of sparse nonnegative least square (SNNLS) solver [5]. Usually 10-20 iterations of SNNLS solver are sufficient to solve these sub problems; however, when a degenerate solution [5] is occasionally detected, we switch to the Interior-Point Newton-like method [6] that is specific to handle these kind of solutions.

Results: Using Matlab 2012b (running on 8 core processor Intel Xeon E5620 @ 2.4 GHz), it took~14 hours to invert QT2R data (size 128 x96x1) using L1-norm solver; while it took ~2.5 hour for L2-norm solver.

Simulation: Fig.1 shows the simulated and reconstructed MWF maps at various SNR. Methods using L1- and L2-norm are visually superior to the conventional approach with reduced mean square error (MSE) and lower symmetric Kullbeck-Leibler (SKL) score (lower score means better match between distributions) for both surrounding matrix and simulated lesions. For high SNR data, both L1-norm and L2-norm solver return similar results with good match to the simulated data. However, at low SNR, L2-based solver is both visually and numerically better than L1-based solver with lower SKL score and lower MSE of myelin water fraction.

Experiment: The MWF map using L1-norm solution is similar to L2-norm for this moderately high SNR (~200) QT2R data. The average symmetric Kullbeck-Liebler score (SKL), commonly used to calculate the divergence between two distributions normalized to 1, between these two solutions is calculated to be 0.34. This low averaged SKL score indicates a good match between T2-distribution maps from L1- and L2-solvers.

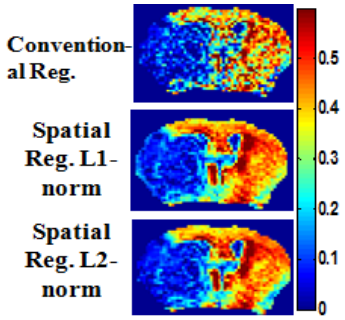

Fig.2. Comparison of MWF maps obtained using the conventional vs L2-norm based vs L1-norm based algorithms. Note the improved detection of myelin water compartment and reduced spatial variability of the L2-norm and L1-norm based algorithms. Results from L1- solver and L2-solver appear similar.

Conclusions: Our preliminary results show that though L1-norm and L2-norm based solver returns similar solution at high SNR (> 200); L2-norm based solver may be more efficient in imposing similarity constraints at low SNR (<200). So, L2-norm solver could be our preferred solution for imposing similarity constraints in context of QT2R analysis.

In near future, we would investigate other possible "hybrid" filter to improve the performance of L1-norm filter at low SNR (100-150) and to test if we can get comparable solutions at low SNR even using L1-norm solver.

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