

Characterization of Intrinsic Susceptibility Gradients Using $R_{1\rho}$ Dispersion

John Thomas Spear^{1,2} and John C. Gore^{2,3}

¹Physics & Astronomy, Vanderbilt University, Nashville, TN, United States, ²Institute of Imaging Science, Vanderbilt University, Nashville, TN, United States,

³Biomedical Engineering, Vanderbilt University, Nashville, TN, United States

Target Audience: Scientists interested in characterization of magnetically inhomogeneous tissues and the effects of intrinsic susceptibility gradients on MRI signals.

Purpose: Spin-lattice relaxation in the rotating frame ($R_{1\rho}=1/T_{1\rho}$) typically reports on slow molecular rotational and translational motions as well as chemical exchange processes.^{1,2} However, in magnetically inhomogeneous media, diffusion through susceptibility gradients also affects $R_{1\rho}$ and appears similar to a slow-exchange process.³ We recently derived a theoretical closed form approximate expression, $R_{1\rho} = \frac{\gamma^2 g^2 D}{(q^2 D)^2 + \omega_1^2}$, to describe these effects on the dispersion of $R_{1\rho}$ with locking field, and demonstrated its ability to quantify the average spatial distribution of inhomogeneities in approximately periodic structures.⁴ Here, γ is the gyromagnetic ratio, g^2 is the mean squared gradient, D is the self-diffusion coefficient, q is the spatial frequency of susceptibility gradients, and ω_1 is the strength of the applied RF field. The average spatial distribution of inhomogeneities is summarized by the correlation time ($\tau_c = 1/q^2 D$), which is the time it takes for a spin to diffuse a characteristic internal dimension, the scale of the gradients. To further substantiate that diffusion is responsible for the observed dispersion of $R_{1\rho}$ at low locking fields, we have run finite-difference simulations of the modified Bloch equations to model diffusion through susceptibility gradients for various structures, and compared the simulated results to experimental data. These effects also occur in the presence of chemical exchange, though (usually) at lower frequencies, and may be separable either because they introduce additional inflections or lower the mean exchange rates obtained from the appropriate fitting. The method has also been used to study and blood samples in vitro.

Methods: Packed sphere structures with varying radii and volume fractions were generated in MATLAB with corresponding ΔB_0 fields calculated everywhere in between the spheres with the assumption that the volume magnetic susceptibility was -8.21×10^{-6} (that of polystyrene) as shown in figure 1. Water was assumed to diffuse between the spheres with coefficient $D = 2.5 \mu\text{m}^2/\text{ms}$ without permeating into the spheres. Bloch equations with transverse RF were calculated for every voxel at each time step in increments of $2 \mu\text{s}$ with the local gradient caused by the surrounding structure incorporated into the chemical shift term. This allows the susceptibility effects to be incorporated directly into the Bloch equation calculation without adding additional steps. Simulations were run for structures with volume fractions varying from 30-60%, each with radii of $5 \mu\text{m}$, and structures with radii varying from 5-15 μm , each with a volume fraction of 60%. Cylindrical structures were also generated and studied to simulate the effect the structure has on $R_{1\rho}$ relaxation. Correlation times were calculated for all structures and the packed sphere structure results were compared to existing experimental data.

Results: The simulated dispersion curves behaved as predicted by theory. Dispersion magnitudes and correlation times increased radii (figure 2). Cylindrical structures showed similar behavior, and the correlation times for both spheres and cylinders are plotted for varying volume fractions and radii in figure 3. The correlation times match well with existing experimental data from samples of packed polystyrene microspheres shown in figure 4.

Discussion: The increase in correlation time with sphere radius at constant volume fraction signifies an increase in the sizes of the space between the spheres or cylinders in the structure. Correlation times increase at a slower rate for cylinders because they are effectively only two-dimensional. The decrease of correlation time with volume fraction corresponds to the decrease in spacing between inhomogeneities.

Conclusion: Diffusion through internal field gradients causes dispersion in $R_{1\rho}$ to a degree that depends on the geometry and sizes of the inhomogeneities responsible for the gradients. The dispersion occurs over low locking fields compared to chemical exchange effects. The inflection frequency can be used to derive the correlation time that describes the average spatial dimensions of the inhomogeneities.

References: [1] Jin et al. Magn Reson Med 65:1448-1460, 2011. [2] Cobb et al. Magn Reson Med 66:1563-1571, 2011. [3] Hills et al. Magn Reson Img 8:321-331, 1990. [4] Spear et al. Magn Reson Med, doi 10.1002/mrm.24837.

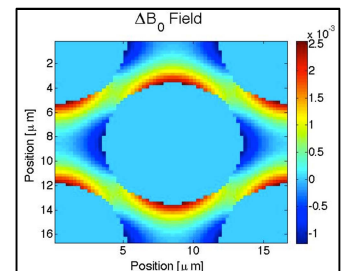


Fig 1. Map of calculated field offsets [Gauss] for spherical unit cell structure with radii= $5 \mu\text{m}$ and volume fraction of 45%.

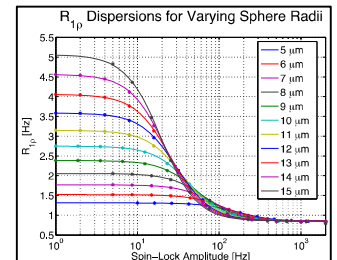


Fig 2. Simulated $R_{1\rho}$ -dispersion curves for spherical structures of varying radii. Note the change in dispersion magnitude and the shift of inflection point.

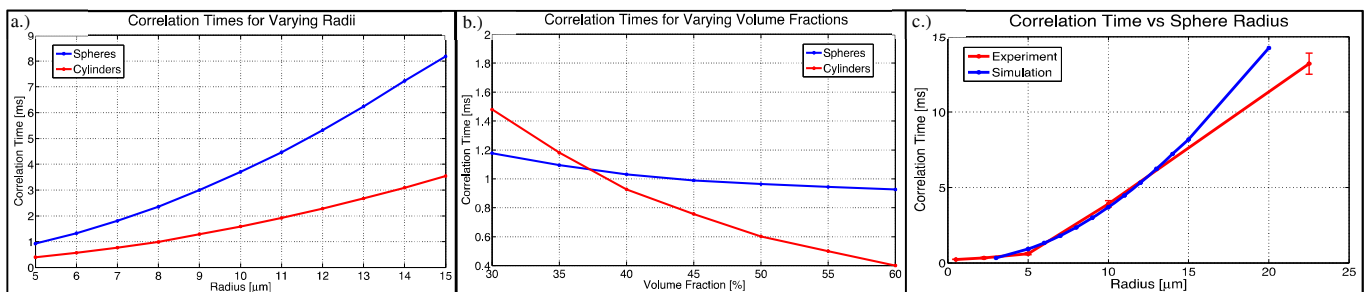


Fig. 3 a.) Correlation time plots with varying radii for both spherical and cylindrical structures. b.) Correlation time plots with varying volume fraction for both spherical and cylindrical structures. c.) Comparison of correlation times from polystyrene microsphere experiments to simulations shows the theory agrees well with small radii.