Tracer-kinetic field analysis in DCE-MRI and DSC-MRI: the reconstruction problem

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TARGET AUDIENCE: Physicists

INTRODUCTION: Tracer-kinetic field theory explicitly models the exchange of indicator between voxels, and its relation to the spatial structure of the concentrations [1-4]. As such it promises to eliminate systematic errors due to large-vessel contamination and bolus dispersion [5,6], and may enable a measurement of additional parameters [7]. The purpose of this study is to propose an algorithm for the inverse problem, and perform an initial evaluation with simulated data.

METHODS: In a tracer-kinetic field model with Q compartments, the measured tissue concentration $C(\mathbf{r},t)=\Sigma_q v_q(\mathbf{r})c_q(\mathbf{r},t)$ is a function of space and time that is fully determined by volume fractions $v_q(\mathbf{r})$, flows $\mathbf{f}_q(\mathbf{r})$, diffusivities $D_q(\mathbf{r})$, and volume transfer constants $k_{pq}(\mathbf{r})$ of the individual compartments (q,p = 1,…,Q). The inverse problem aims to reconstruct these fields at the voxel locations from measurements of $C(\mathbf{r}_i,t_k)$ on a 3D slab of $N = N_xN_yN_z$ voxels at positions $\mathbf{r}_i$ measured at N_t times $t_k = k\Delta t$.

Algorithm: After forward time discretization of the field equations, and space discretization by integration over each voxel, the concentrations $c_q(\mathbf{r}_i,t)$ in each compartment can be organized into an NQ-vector $\mathbf{c}(t)$. The result is a large sparse matrix equation:

$$\mathbf{c}(t+\delta t) = \mathbf{c}(t) - \delta t\, \mathbf{v}^{-1}\mathbf{k}\mathbf{c}(t) - \delta t\, \mathbf{v}^{-1}\mathbf{k}_b\mathbf{c}_b(t) \qquad (1)$$

Here $\mathbf{c}_b(t)$ are the boundary concentrations in the voxels adjacent to the imaged slab, $\mathbf{v} = \mathrm{Diag}\{v_q(\mathbf{r}_i)\}$, and $\mathbf{k}$, $\mathbf{k}_b$ are sparse matrices that depend on $\mathbf{f}_q(\mathbf{r}_i)$, $D_q(\mathbf{r}_i)$, $k_{pq}(\mathbf{r}_i)$. Given the initial condition $\mathbf{c}(0)=\mathbf{0}$, and initial values of the parameters $\mathbf{P} = \{\mathbf{v}, \mathbf{k}, \mathbf{k}_b, c_b(t_k)\}$, Eq (1) predicts $\mathbf{C}(t_k)$ at any $t_k < T_{acq}$. The prediction is optimized by adjusting $\mathbf{P}$ using a gradient-descent algorithm. The problem is solved sequentially for $T_{acq}=\Delta t, 2\Delta t, \dots, N_t\Delta t$ -- each time using the solution of the previous iteration as initial values for the next.

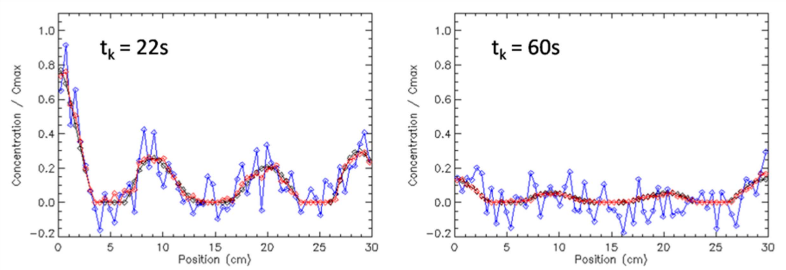

Fig 1. Measured (blue), exact (black), and reconstructed (red) concentrations at acquisition times of 22s (left) and 60s (right).

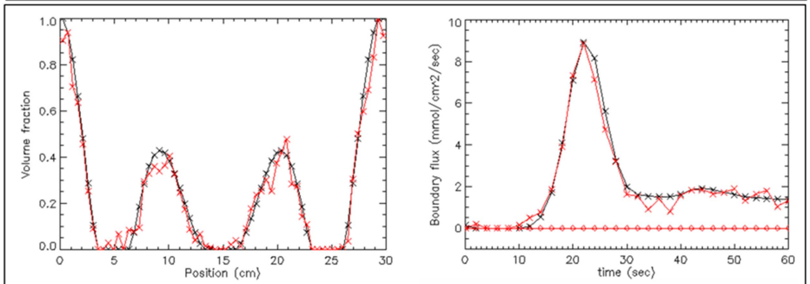

Fig 2. Left: Exact (black) and reconstructed (red) volume fractions (left). **Right**: exact (black) and reconstructed (red) arterial input function.

Simulations: As an initial test, the approach is applied to reconstruct the simplest field model, a one-compartment convective model with parameters $v_p(\mathbf{r})$, $\mathbf{f}_p(\mathbf{r})$ satisfying $\nabla\cdot\ \mathbf{f}_p(\mathbf{r}) = 0$. A one-dimensional arrangement of N=64 voxels is simulated, with voxel size 5mm, Δt=2s, N_t=30 acquisitions, and Gaussian noise with a high SD of 10% of the peak concentration. The flow f_p is 1.5 ml/s/cm^2; $v_p(x)$ varies from 0 to 1 as shown in fig 2 (left); and $c_b(t)$ is taken from an experimental population-average [8] as shown in fig 2 (right).

RESULTS: Fig 1 shows the exact, measured and reconstructed $C(x_i,t_k)$ at two different t_k. Fig 2 shows the exact and reconstructed $v_p(x_i)$ and $c_b(t_k)$. The error in the reconstruction of v_p, f_p and c_b is 1.8%, 16% and 2.7%, respectively. Reconstructed concentrations are accurate to within 1.9%. Calculations take 2.1 min on a standard laptop PC.

CONCLUSION: The spatial correlations between the voxels impose additional constraints on the solution that enable accurate reconstruction even at high noise levels. They also enable a reconstruction of the AIF, which may have significant implications in terms of accuracy and practicality. The calculation times are reasonable in the small problem but will rise exponentially at 3D and in two-compartment models. On the other hand, the algorithm has not been optimized for efficiency, and a reconstruction of the entire imaging slab is not usually required. Future studies will focus on a more in-depth analysis of the stability and practicality of the approach in multi-compartmental systems.

REFERENCES **[1]** Thacker et al. JMRI 2003; 17: 241-255 **[2]** Pellerin et al MRM 2007; 58: 1124-34 **[3]** Christensen JMRI 2008; 27:1371-81 **[4]** Koh et al MRM 2013; 69: 269-76. **[5]** Le Bihan and Turner MRM 1992; 27:171-178 **[6]** Calamante MRM 2000; 44:466-73 **[7]** Frank et al MRM 2008; 60:1284-91 **[8]** Parker et al. MRM 2006; 56: 993-1000.