Enforcing divergence free to velocity data from 4D flow MR images

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Introduction: 4D flow imaging has shown great potential to study different cardiovascular diseases. However, the velocity data provided by 4D flow is highly prone to be affected by respiratory motion due to the long acquisition time, inhomogeneities due to large FOV, and partial volume effects due to low spatial resolutions, among others. Some of these terms can be corrected by applying a linear phase correction estimated from static tissues; however, a great amount of inaccuracies may still remain. Another way to correct the data is to assume that blood flow is incompressible and to enforce the velocity field to be divergence free. Lately, some methods have been proposed to enforce divergence free to the velocity field [1,2]. In this work, we propose a novel method to enforce divergence free to the velocity data by calculating a velocity corrector for each pixel in the domain of interest.

Method: Theory: To enforce divergence-free to a 3D velocity field, VI, our algorithm tries to find a corrector, Vc , so that the divergence of VI + Vc is equal to 0 in a certain domain of interest. To find an optimal corrector we minimize the following objective function:

$$J_\alpha(V_C) = \frac{1}{2}\int_\Theta (\text{div } (V_C + V_I))^2\, dx + \frac{\alpha}{2}\|V_C\|_w^2$$

The second term involves a regularity factor, where α is defined as a positive and real value and the norm □V_C □_w^2 is introduce to avoid large numbers of VC. It can be shown that previous equation is Fréchet-derivable, and hence it can be possible to obtain an optimal condition, which can be written as a variational formulation for the corrector Vc, which read as:

$\int_\Omega \text{div}(V_I + V_C)\text{div } w\, dx + \alpha\left[\int_\Omega (V_c \cdot w\, dx + \nabla V_C : \nabla w) dx\right] = 0,$ were Ω represents the domain of interest.

The formulation was integrated into a finite element solver (FreeFem++) which finds an optimal corrector for each pixel in the domain of interest.

Experiments: To showcase the applicability of the method we obtained velocity data from 2 volunteers using a 4D flow scan of the whole heart. Using the 4D flow data, the aorta was segmented during a representative systolic phase. The segmented aorta represented our volume of interest, which was then used to create a tetrahedral mesh necessary for the finite-element solver. The velocity data in the domain of interest was corrupted by adding Gaussian noise with a mean value of 0.035% of the maximum velocity value. In order to find an optimal corrector, we ran the finite element solver for different values of α (0, 0.1, 0,5, 1, 10, 20 , 30, 50 and 100). From these solutions, we obtained different sets of correctors. We calculated the divergence of the corrected velocity field (J1) and also the sum of the magnitude of each corrector in the entire domain (J2).

$$J1 = \frac{1}{2}\int_\Theta (\text{div } (V_C + V_I))^2\, dx \qquad J2 = \frac{1}{2}\|V_C\|_w^2$$

Results: Figure 1 shows the in plane velocity vectors of the data which is corrupted (1.a) and then corrected using two different values of α. The J1 values of the starting corrupted velocity field was equal to 5.22x108, which is reduced by any value of α lower than 100. In figure 2, we analyze the influence of α in the final values of the divergence, which shows an asymptotically behavior of the divergence for α greater than 10. Figure 2 also shows the relation between the J1 and J2. From these figures it can be observed that a good correction of the divergence can be achieve by values of α lower than 10, however for values lower than 10, the value of the corrector increases drastically and therefore a values between 10 and 100 should be selected. The mean time that the solver took to obtain a corrector for a specific α value was 1 minute in a computer -Intel Core i5, CPU 2.50 GHz, RAM 8 GB, 64 bits processor.

Discussion: We have proposed a novel methodology that allows us to reduce the divergence of a velocity field provided by 4D flow imaging. The method proposed a close solution, which was implemented on a finite element solver to find a corrector in the entire domain of interest within a few seconds.

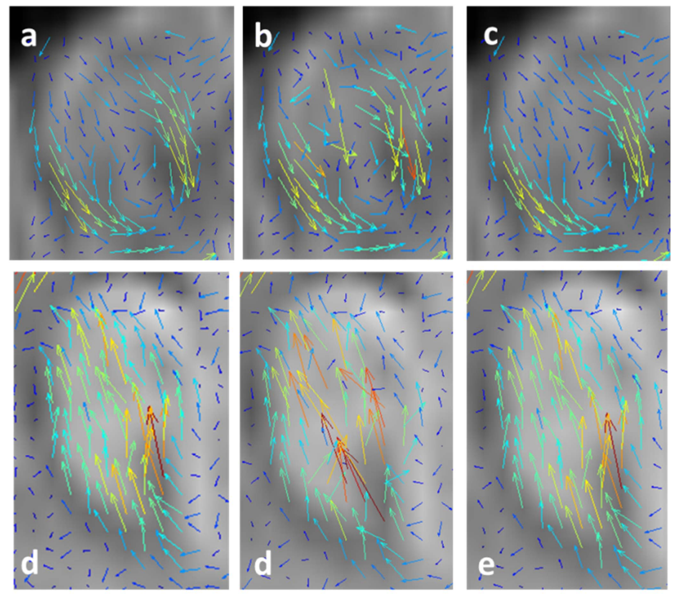

Figure 1. Velocity field data corrupted by noise (a,d) and velocity corrected data for 2 different values of α. In b,c, α=0.5 and in c,d α=10 .

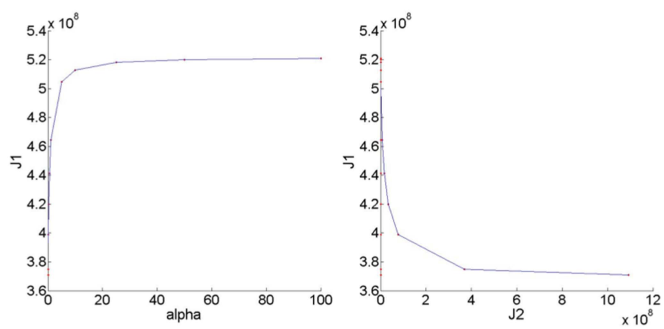

Figure 2. J1 values (left) of the velocity corrected field for different values of α. On the right, we plot J1 versus J2. From this plot we can select the value of α close to the origin since minimizes the divercence and a mimimun values of the corrector.

References:[1] Busch, J. et al, Magnetic Resonance in Medicine, 69:200-210, (2013).[2] Ong, F. et al, Journal of Cardiovascular Magnetic Resonance, 15(Suppl 1):E26, (2013).