A phase constrained reconstruction method in compressed sensing

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Introduction: Half Fourier acquisition has widely been used for a long time to shorten the measurement time in MRI. Due to spin echo formation, image phase in turbo spin echo sequences can be estimated by using fully sampled *k*-space data of only low spatial frequencies. Phase correction methods[1, 2] were developed to synthesize the missing data. The combination of half Fourier acquisition with compressed sensing has also been investigated[3, 4]. One drawback of these methods is that they totally replace the phase of the resulting images by an estimate. However, the estimated phase is usually not perfectly accurate due to frequency truncation, which may lead to amplified artifacts in the reconstructed image. In this work, we introduce a relaxed phase constraint term in the regularized reconstruction, to encourage the phase consistency between the result and the estimation, instead of an overall phase replacement in the compressed sensing reconstruction with half Fourier acquisition.

Method: In the proposed nonlinear reconstruction, the cost function is $f = \|U(Fx - y)\|_2^2 + \lambda_{l1}\|\psi x\|_1 + \lambda_{TV}TV(x) + \lambda_{pc}R_{pc}$. Here x is the intermediate image in each iteration; y is the acquired *k*-space data; F is the Fourier transform operator; U represents the *k*-space sampling mask with binary entries; ψ is a sparsifying transform operator (e.g. wavelet); TV is the total variation operator; R_{pc} is the proposed phase constraint term. To clarify the requirements for such a phase constraint term, consider the following facts: a) phase estimation obtained from whatever ways, may present good accuracy in some regions, but provide poor accuracy in other regions, and usually the exact distribution of the estimation error is unknown; b) To avoid the overall replacement of phase information as in typical Homodyne or POCS based methods, the phase constraint term here is required to automatically promote the consistency in regions with good estimation, but allow exceptions in regions with bad estimation, even though the exact distribution of the estimation error is unknown. We propose a regularization term, which approximately meets these requirements: $R_{pc} = \|g\|_1 = \left\|\left(x \circ e^{-iP} - |x|\right) \circ |x|\right\|_1$. Here, P is the estimated phase; $A \circ B$ represents the Hadamard product of A and B. The solution is obtained by minimizing the cost function f with the nonlinear conjugate gradient method. The gradient of R_{pc} is: $\nabla R_{pc} = \left(\frac{\partial R_{pc}}{\partial x}\right)^* = \left(\frac{\partial R_{pc}}{\partial g}\frac{\partial g}{\partial x} + \frac{\partial R_{pc}}{\partial \bar{g}}\frac{\partial \bar{g}}{\partial x}\right)^*$. Here $\frac{\partial R_{pc}}{\partial g} = \left[\frac{\partial R_{pc}}{\partial g_1}, \ \dots \frac{\partial R_{pc}}{\partial g_n}\right]$ with $\frac{\partial R_{pc}}{\partial g_k} = \frac{1}{2}\ \frac{\overline{g_k}}{|g_k| + \varepsilon}$; ε is a positive small parameter to avoid 'division by zero'; $\frac{\partial g}{\partial x}$ is a diagonal matrix with entries $\left(\frac{\partial g}{\partial x}\right)_l = \frac{3|x_l|e^{-iP_l}}{2} - \bar{x}_l$; $\frac{\partial R_{pc}}{\partial \bar{g}} = \overline{\left(\frac{\partial R_{pc}}{\partial g}\right)}$; $\frac{\partial \bar{g}}{\partial x}$ is a diagonal matrix with entries $\left(\frac{\partial \bar{g}}{\partial x}\right)_l = \frac{\bar{x}_l \bar{x}_l e^{iP_l}}{2(|x_l| + \varepsilon)} - \bar{x}_l$. The proposed reconstruction method was first evaluated using a digital phantom with both smooth and rapid phase variation (Fig1 a~b). Full k-space data was obtained by FFT, and undersampled by the pattern shown in Fig1c (acceleration factor = 5.0). The phase estimate was obtained from the fully sampled region in the *k*-space center, which was used in both the POCS incorporated method[4] and our proposed method. The reconstruction errors of three different methods were compared: wavelet + total variation; wavelet + total variation + POCS; wavelet + total variation + the proposed phase constraint. The sampling pattern in the phantom experiment has also been implemented into a 3D turbo spin echo sequence. Volunteer data were acquired with a single channel CP knee coil on a 3.0T clinical MR scanner (MAGNETOM Trio, Siemens, Erlangen) with following parameters: *k*-space matrix size [256 x 256 x 176], TR/TE =1300ms/44ms, echo train length = 42, acceleration factor = 3.5. All image reconstructions were performed in Matlab.

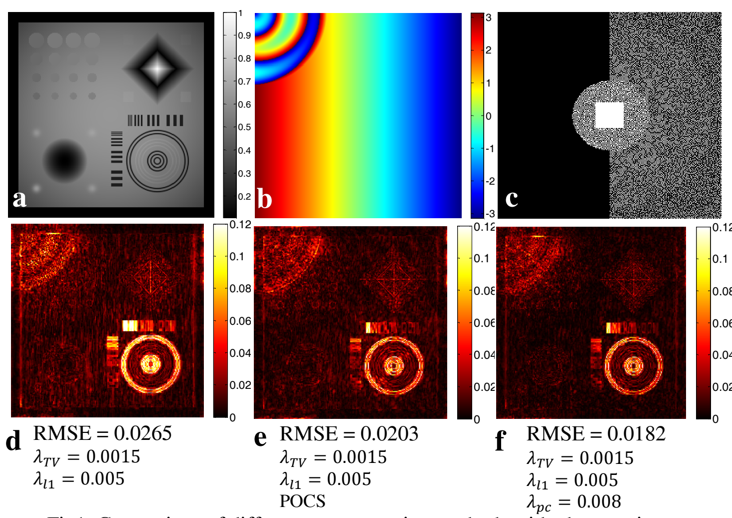

Fig1. Comparison of different reconstruction methods with phantom image

Results & Discussion: It shows that POCS incorporated reconstruction (Fig1e) reduced the error level by applying direct phase correction in the iterations compared to the normal compressed sensing reconstruction (Fig1d). However the same suboptimal phase estimation was repetitively applied to the intermediate results, which hinders the convergence of the iterative reconstruction towards the right direction. In the proposed method, the estimated phase is just utilized as a tool to sparsify the image difference between the phase corrected result and its magnitude in the iterative reconstruction. The $l1$ norm of the image difference tolerates sparsely distributed estimation errors, meanwhile enforces the consistency of the estimated and the reconstructed phase in regions with good accuracy by pursuing the sparseness of the difference. This explains why the proposed method generally presented even lower errors in regions with smooth phase variation, but did not produce higher errors in regions with rapid phase variation compared to the POCS incorporated reconstruction. In the in vivo experiments, it shows the proposed method recovered more image details compared to the normal compressed sensing reconstruction. Fine details like the trabecular structure within the bone appear much clearer and the images look markedly 'crisper' especially in regions with smooth phase variation (Fig2). The difference term g (Fig2d) shows the estimation errors were mainly located in regions with rapid phase variation (Fig2c).

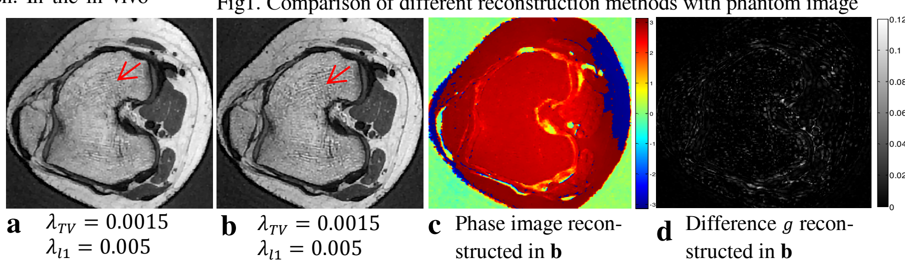

Fig2. Comparison of different reconstruction methods with in vivo data

Conclusion: A phase constraint term is proposed to replace the direct phase correction methods in compressed sensing reconstruction with half Fourier acquisition. Both simulation and in vivo experiments showed the super performance of the proposed method.

References: 1. Nell, D.C., IEEE Trans. Med. Imaging, 10, 154-163, 1991; 2. EM Haccke, J Magn Reson 1991;92:126–145; 3. Lustig M et al, MRM 2007, 1182 -1195; 4. Doneva M et al, ISMRM 2010: 485;