3D GlObally Optimal Surface estimation (3D-GOOSE) algorithm for fat and water separation

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Target audience: Researchers and clinicians that work with applications of the fat-water separation in MRI data.

Purpose: To introduce a robust algorithm that is capable of utilizing the smoothness of the field-map in three spatial dimensions to resolve the ambiguities in fat-water decomposition. Current fat-water decomposition algorithms that rely on field map smoothness may still be sensitive to local minima effects, especially in challenging cases with considerable variations in field map. We had recently introduced a novel graph cut algorithm termed as GlObally Optimal Surface Estimation (GOOSE) that is guaranteed to provide the global minimum of a smoothness constrained optimizing formula. This scheme was observed to considerably improve the performance on challenging datasets. However, our previous implementation was restricted to two dimensional images and hence was not capable of exploiting the smoothness of the field between slices. The main focus of this work is to extend the GOOSE algorithm to 3-D, thereby exploiting the three dimensional smoothness of the field-map and further improve the robustness.

Methods: In multi-echo water and fat decomposition, a sequence of images is collected with different echo time (TE) shifts. The signal at each individual pixel is described by the Dixon model [1]:

$$s(\mathbf{r},t_n) = (\rho_{\text{water}}(\mathbf{r}) + \rho_{\text{fat}}(\mathbf{r})[\sum_{i=1}^{M}\beta_i e^{j2\pi\delta_i t_n}])e^{-i\gamma(\mathbf{r})t_n}, n = 1,..,N \qquad (1)$$

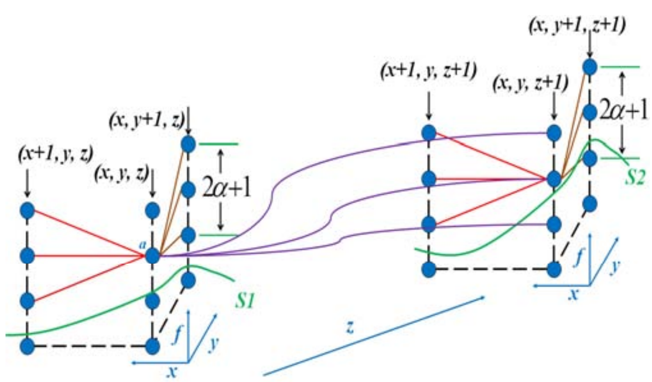
Classic methods focus on solving the unknowns concentrations ($\rho_{\text{water}}(\mathbf{r})$ & $\rho_{\text{fat}}(\mathbf{r})$) and magnetic inhomogeneity values ($\gamma(\mathbf{r})$) by providing analytical or non-linear optimizing solution, which often fail due to the non-linearity of the equation, presence of multiple feasible solutions, presence of noisy pixels, and the large range of field map. A common approach to minimize the ambiguities is to exploit the smoothness of the field inhomogeneity map by posing the problem as a smoothness penalized optimization scheme. Current algorithms (e.g. region growing, region merging or iterative graph cut) provide good solutions in many cases, but fail to provide the global minimum in more challenging cases. We propose to extend our 2-D GOOSE algorithm to 3-D to exploit the smoothness of the field map in three dimensions. We discretize the field-map value at each location to a uniform grid $\boldsymbol{R}_f = n\Delta$, Δ is the grid spacing, while n is the discrete index. The estimate of the field-map is formulated as the constrained optimization scheme:

$$\hat{f} = \arg\min_{f(\mathbf{r})} \sum_{\mathbf{r}} C(\mathbf{r}, f(\mathbf{r})) \quad \text{such that} \quad \begin{aligned} f(\mathbf{r}+\mathbf{e}_x) - f(\mathbf{r}) &\le |F| \\ f(\mathbf{r}+\mathbf{e}_y) - f(\mathbf{r}) &\le |F| \\ f(\mathbf{r}+\mathbf{e}_z) - f(\mathbf{r}) &\le |F| \end{aligned} \qquad (2)$$

Here, the criterion $C(\mathbf{r}, f(\mathbf{r}))$ is the likelihood measure for the field map that is obtained using the VARPRO formulation in [2]; $\mathbf{e}_x = (\pm1, 0, 0)$, $\mathbf{e}_y = (0, \pm1, 0)$ and $\mathbf{e}_z = (0, 0, \pm1)$ are the unit vectors in the x, y, and z (slice) directions. The difference between field map values at adjacent voxels is constrained less than $F = \alpha\Delta$ to minimize the ambiguities, α is the smoothness constraint. We solve (2) using the globally optimal surface search algorithm, introduced in [3] (Fig. 1). The algorithm is theoretically guaranteed to provide the global minimum of (2). Once the optimal field map is obtained, the fat and water concentrations at each pixel can be determined as $\rho = (\mathbf{A}^T_f\mathbf{A}_f)^{-1}\mathbf{A}^T_f\,\mathbf{s}$, where $\mathbf{s}$ is the vector of measurements and the matrix $\mathbf{A}_f$ models the equation (1).

Results: The implementation uses six spectral fat peaks at δ = [-3.80, -3.40, -2.60, -1.94, -0.39, 0.60] ppm, with relative weight β = [0.0870, 0.6930, 0.1280, 0.0040, 0.0390, 0.0480]. The search range $\boldsymbol{R}_f$ of [$-2\delta_{\max}$, $2\delta_{\max}$] has been shown sufficient to account for field imhomogeneity in all datasets. We validated the proposed algorithm using the metric introduced in the 2012 ISMRM Challenge for developing fat water separating algorithms(http://www.ismrm.org/challenge/). We observed that the 3-D scheme provides quantitative scores that are marginally better than the 2-D scheme (around 10 points/1000 on average). This is encouraging, considering that the 2-D scheme is already performing well with an average score of 9927/10000. We also observe from the fat water images that the 2-D GOOSE scheme yielded a small swap in one of the datasets, which may be misinterpreted as a cyst. It can be seen in Fig.2 that the proposed 3-D scheme is capable of resolving the swap by exploiting the smoothness of the field map between slices.

Discussion and Conclusion: A 3-D graph cut fat-water separating scheme is proposed in this work. The scheme demonstrated robustness in challenging datasets. Computation complexity can be further reduced.



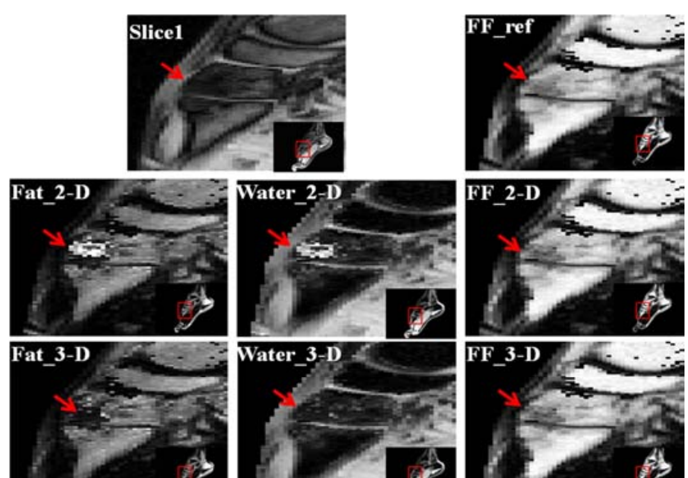


Figure 1 (Left): Multi optimal graph surfaces search algorithm model. At each slice, a 3-D graph (x, y, f) is built in which node ***a*** is connected by edges to its neighbors in both x and y directions. ***a*** is also connected to neighbors in adjacent slices. Edge connectivity exists between ***a*** and its neighbors only in the range of $2\alpha + 1$. An optimal surface is obtained in 3-D graph at each slice (***S1*** and ***S2***). **Figure 2** (Right): Fat-water separating results using both 2-D and 3-D on foot (saggital) dataset, with TEs = {1.4, 3.7, 4.4, 7.5, 10.6} ms. Note that this dateset is non uniformly sampled. Top row: (Left) The original image of the first slice; (Right) **F**at **F**raction map for **ref**erence provided by 2012 ISMRM Challenge organizers (FF_ref); Second row: Separating result using 2-D graph surface search. A fat-water swap can be seen in both Fat_2D (separated **Fat** signal using **2-D** GOOSE scheme) and Water_2D. Third row: Separating result using 3-D graph surface search. The swap in 2-D is removed in 3-D scheme. It can be observed that the swap is not reflected in fat fraction maps.

References: [1] H. Yu, A.Shimakawa, C.A. McKenzie, E. Brodsky, J. H. Brittain, and S. B. Reeder, Magnetic Resonance in Medicine, 2008.
[2] D. Hernando, P.Kellman, J.P.Haldar, and Z.P.Liang. Magnetic Resonance in Medicine, 2008.
[3]X. Wu and D. Chen, Automata, Languages and Programming, 2002.