Sorted Compressed Sensing in MRI

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Target Audience: MR mathematicians, physicists and engineers, and clinicians who want to deepen their understanding of undersampled MRI reconstruction techniques.

Introduction: In Compressed Sensing (CS), setting the regularization parameter λ, i.e. the trade-off between data consistency and penalization, has been a widely studied topic in MRI. Nowadays, λ is a fixed number that penalizes the whole reconstructed image. However, the current approach is not always accurate in MRI. To improve the traditional framework for image reconstruction, we propose a CS technique with variable weight, which penalizes the pixels of the recovered image according to their magnitude [1]. Pixels are sorted in descendent order according to its magnitude and are penalized with a non-increasing sequence of regularization parameters. The main contribution of this work is in high order images, e.g. volumetric brain images, where choosing a variable weight may lead to poor sparse representation, when the ideal image is sparse [1]. Herein, we present quantitative susceptibility map (QSM) reconstructions in in-vivo data, where the Sorted Compressed Sensing (SCS) produced results that demonstrate it is feasible to reconstruct high quality images. The proposed method produced gains up to 3-4 dB with respect of traditional CS.

Theory: To quantify tissue magnetic susceptibility, $\boldsymbol{\chi}$ maps, the system of linear equations: $\mathbf{b} = \mathbf{F}^{-1}\mathbf{D}\mathbf{F}\boldsymbol{\chi} + \mathbf{e}$ must be solved; where $\mathbf{b} \in \mathbb{R}^P$ is the normalized field map, $\mathbf{D}$ is the susceptibility kernel in *k*-space, $\mathbf{F}$ is the Fourier transform operator and $\chi \in \mathbb{R}^N$ is the susceptibility vector and $\mathbf{e} \in \mathbb{R}^P$ is the acquisition noise [2]. This is an ill-posed problem, because $\mathbf{D}$ undersamples the measured field [3]. In a SCS framework, $\boldsymbol{\chi}$ map recovery is as follows:

$$\hat{\chi} = \arg\min_{\chi \in \mathbb{R}^N} \frac{1}{2} || \mathbf{F^{-1}DF}\chi - \mathbf{b} ||_{\ell_2}^2 + \|\Lambda\Psi\chi\|_{\ell_1},$$

where Λ is a diagonal matrix with the sorted regularization parameters on its diagonal, i.e. $\Lambda_{k,k} = \lambda_k$; and $\boldsymbol{\Psi}$ is a wavelet transform operator that includes the sorting function. To set the sequence λ_k with $k \in \{1, \dots, P\}$, we first may consider the Benjamini-Hochberg (BHq) procedure [4]: $\lambda_k = \lambda_{BH}(k) = \Phi^{-1}\left(1 - \frac{k\theta}{2\xi}\right)$, where $\Phi^{-1}(n)$ is the nth quantile of the standard normal distribution, θ is a fixed parameter in $[0,1]$ and ξ is the number of non-zero coefficients of $\Psi\chi$. However, this approach can be improved by considering the increase in variance of the model [1]. Therefore, the sequence begins with $\lambda_1 = \lambda_{BH}(1)$ and then, for $k > 1$, we compute the corrected values as follows:

$$\lambda_k = \lambda_{BH}(k)\sqrt{1 + \frac{\sum_{j<i} \lambda_{BH}^2(j)}{N - k}}.$$

Methods: To test the proposed framework, we reconstructed the susceptibility χ map from numerical phantom and noisy in-vivo field maps using CS and SCS techniques. For numerical phantom, we defined three regions: gray matter (χ = 0.027 ppm), cerebrospinal fluid (χ = –0.018 ppm) and white matter (χ = –0.023 ppm). In-vivo data was acquired from a healthy young volunteer using a 3D SPoiled Gradient Recalled Echo (SPGR) sequence at 1.5T. 62 axial slices with 2.5 mm slice thickness and FOV of 240×240×155 mm^3 for a TR/TE=58 ms/40 ms, FA=15°, 512×256 in-plane and 12:20 min, with flow compensation [6].

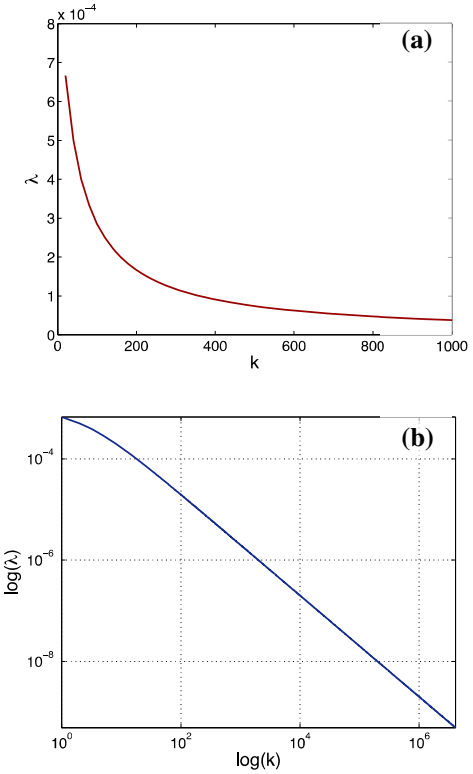

Fig.1: $\boldsymbol{\lambda_k}$as function of k. (a) Presentation of the 1000 largest coefficients of $\boldsymbol{\lambda}$. (b) Logarithmic display of $\boldsymbol{\lambda_k}$.

Results: For numerical susceptibility reconstruction we computed the signal-to-error ratio (SER) of SCS and CS, reporting 27.3 dB and 23.7 dB respectively (numerical χ maps not shown). For in-vivo data we used a field map with 200 iterations of dipole fitting. For image display we present only the first 10^3 sorted coefficients of optimal λ in Fig.1(a). To appreciate the structure of all the coefficients of λ (approx. $4 \cdot 10^6$ coefficients), a log-log plot is presented in Fig.1(b). Fig.2, illustrates the axial view of the χ map reconstructions using CS and SCS respectively. Optimal setting for CS is $\lambda = 2 \cdot 10^{-4}$ and for SCS is λ_k. Preservation of sharp edges can be observed in SCS.

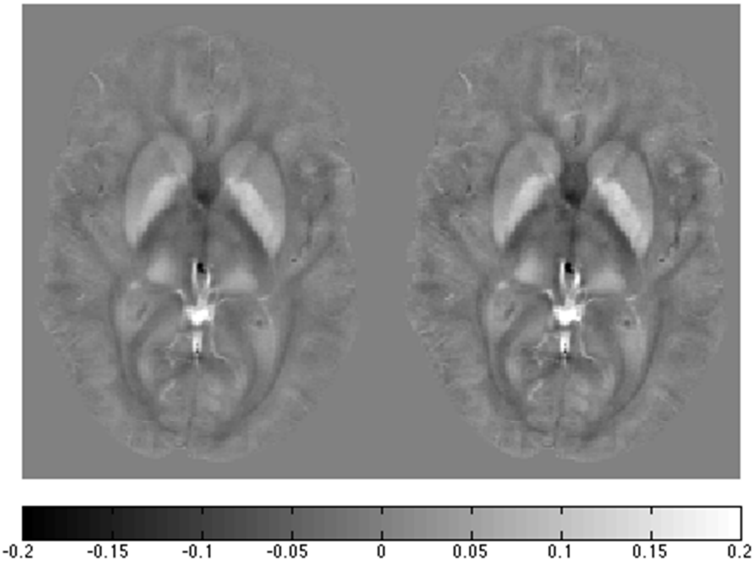

Fig.2: Axial view (in ppm). In-vivo χ map reconstructions via CS (left) and SCS (right).

Conclusion: We have presented a new reconstruction framework tailored for MRI, which demonstrated in the in-vivo dataset an increase in image quality with respect of traditional CS. And in the numerical phantom gains of 3-4 dB were produced by SCS over CS.

References: [1] Bogdan M et al., 2013arXiv1310.1969B [2] Bilgic B et al., ISMRM 2011;746 [3] Fan AP et al., ISMRM 2011;4472 [4] Benjamini Y and Hochberg Y, JRSS; 57(1):289–300 [5] Bilgic B et al., NIMG 2012; 59(3):2625-2635