Highly Accelerated Dynamic Parallel MRI Exploiting Constrained State-Space Model with Low Rank and Sparsity

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Introduction: Fast magnetic resonance imaging (MRI) techniques [1-4], which lead to signal recovery from incomplete data, have been introduced in dynamic imaging to improve spatiotemporal resolution without apparent loss of image quality. In this respect, we propose a novel, highly accelerated dynamic parallel MRI reconstruction method exploiting a constrained state space model with low rank and sparsity while jointly estimating spatiotemporal kernels and missing signals in k-t space in an iterative fashion. Spatiotemporal kernels stacked across multiple time frames are estimated using the low rank constraint due to the nature of smoothly varying spatiotemporal correlation in k-t space during calibration, while the solution is projected onto the reconstructed k-t space with the sparsity constraint imposed on the estimated dynamic images in x-f space. It is expected that compared with conventional methods the proposed technique exhibits superior performance in removing artifacts and noise even at high acceleration factors.

Method: We can formulate a spatiotemporal kernel within the Nyquist sampled region for γ^{th}coil of t^{th} time frame:$\mathbf{y}_{\gamma,t}^{\mathrm{Nyq}} = \mathbf{S}_t^{\mathrm{Nyq}}\mathbf{g}_{\gamma,t}$ **(1)**, where $\mathbf{y}_{\gamma,t}^{\mathrm{Nyq}}$ is the target calibrating signal, $\mathbf{S}_{t}^{\mathrm{Nyq}}$ is the source matrix consisting of neighboring signals along both spatial and temporal directions, and $\mathbf{g}_{\gamma,t}$ is the spatiotemporal kernel. For ease of notation, we henceforth drop the coil index. Assuming that the kernels are independent across time frames, we can extend (1) to the semi-batch type including τ frames among the entire time frames for time set, $T_t \in \{t, t+1, \cdots, t+\tau-1\}$ by creation of block matrices (Fig.1a). We model the kernel sequence using state-space model with the block diagonal structure: $\mathbf{g}_{\mathrm{T_t}} = \mathbf{g}_{\mathrm{T_{t-1}}} + \mathbf{w}$ **(2)**, and $\mathbf{y}_{\mathrm{T_t}}^{\mathrm{Nyq}} = \tilde{\mathbf{S}}_{\mathrm{T_t}}^{\mathrm{Nyq}}\mathbf{g}_{\mathrm{T_t}} + \mathbf{v}$ **(3)**, where $\mathbf{w}$ and $\mathbf{v}$ denote the process and measurement noises with $\mathbf{w} \sim \boldsymbol{\mathcal{N}}(\mathbf{0}, \boldsymbol{Q})$ and $\mathbf{v} \sim \boldsymbol{\mathcal{N}}(\mathbf{0}, \boldsymbol{R})$, respectively. The KF technique provides a solution to the above state-space model, leading to coil-dependent weighted denoising in reconstruction (Fig.1b). However, it is well known that this approach is still sub-optimal as long as the calibration is confined within the Nyquist sampled region in k-space. To overcome the statistical inadequacies, the linear constraint is applied extending the region of calibration from the central to the entire k-space by directly projecting the unconstrained kernel sequence from KF onto the constrained surface: $\mathbf{y}_{\mathrm{T_t}} = \tilde{\boldsymbol{S}}_{\mathrm{T_t}}\mathbf{g}_{\mathrm{T_t}}$ **(4)**. In dynamic cardiac imaging, the temporal signal variation of the image sequence is smoothly varying, which implicitly means that the kernel sequence also result in time-variation kernels only at low temporal frequencies imposing that kernel sequence can be approximated using a few significant singular values. Exploiting the characteristics of image and kernel sequences, we enforce additional two constraints: 1) low rank for kernel sequence; 2) temporal sparsity for image sequence. The proposed constrained KF optimization problem is as follows:

$$\min_{\mathbf{g}_{\mathrm{T_t}}, \mathbf{x}_{\mathrm{T_t}}} \frac{1}{2}\left\|\mathbf{y}_{\mathrm{T_t}} - \tilde{\mathbf{S}}_{\mathrm{T_t}}\mathbf{g}_{\mathrm{T_t}}\right\|_2^2 + \frac{\alpha}{2}\left\|\mathbf{g}_{\mathrm{T_t}} - \hat{\mathbf{g}}_{\mathrm{T_t}}\right\|_{\mathbf{W}_{\mathrm{T_t}},2}^2 + \mathrm{R}\left(\mathbf{g}_{\mathrm{T_t}}, \mathbf{x}_{\mathrm{T_t}}\right) \quad \textbf{(5)},$$

$$\mathrm{R}\left(\mathbf{g}_{\mathrm{T_t}}, \mathbf{x}_{\mathrm{T_t}}\right) = \lambda_{\mathrm{L}}\left\|\mathbf{G}_{T_t}\right\|_* + \lambda_{\mathrm{s}}\left\|F_t F_s \mathbf{x}_{\mathrm{T_t}}\right\|_1 + \frac{\beta}{2}\left\|\mathbf{x}_{\mathrm{T_t}} - \boldsymbol{\mathcal{A}}\left(\mathbf{y}_{\mathrm{T_t}}, \mathbf{g}_{\mathrm{T_t}}\right)\right\|_{,2}^2 \quad \textbf{(6)},$$

where $\mathbf{G}_{T_t}$ is a kernel sequence matrix given by $\mathbf{G}_{T_t} = [\mathbf{g}_t\, \mathbf{g}_{t+1} \cdots \mathbf{g}_{t+\tau-1}]$, $\mathbf{x}_{\mathrm{T_t}}$ is a reconstructed k-space, $\boldsymbol{\mathcal{A}}(\cdot)$ is a reconstruction operator, and $\mathbf{W}_{\mathrm{T_t}}$ is a weighting matrix setting to $\left(\mathbf{P}_{T_t}^{+}\right)^{-1}$. Under the framework of alternating minimization algorithm, the cost function is separated into two-step scheme, which is convex with respect to one variable while keeping others fixed:

$$\mathbf{g}_{T_t}^{(n)} = \min_{\mathbf{g}_{\mathrm{T_t}}} \frac{1}{2}\left\|\mathbf{y}_{\mathrm{T_t}} - \tilde{\mathbf{S}}_{\mathrm{T_t}}\mathbf{g}_{\mathrm{T_t}}\right\|_2^2 + \frac{\alpha}{2}\left\|\mathbf{g}_{\mathrm{T_t}} - \hat{\mathbf{g}}_{\mathrm{T_t}}\right\|_{\mathbf{W}_{\mathrm{T_t}},2}^2 + \lambda_{\mathrm{L}}\left\|\mathbf{G}_{T_t}\right\|_* + \frac{\beta}{2}\left\|\mathbf{x}_{T_t}^{(n-1)} - \boldsymbol{\mathcal{A}}\left(\mathbf{y}_{\mathrm{T_t}}, \mathbf{g}_{\mathrm{T_t}}\right)\right\|_2^2 \quad \textbf{(7)}$$

$$\mathbf{x}_{T_t}^{(n)} = \min_{\mathbf{x}_{\mathrm{T_t}}} \left\|F_t F_s \mathbf{x}_{\mathrm{T_t}}\right\|_1 + \frac{\beta}{2\lambda_s}\left\|\mathbf{x}_{\mathrm{T_t}} - \boldsymbol{\mathcal{A}}\left(\mathbf{y}_{\mathrm{T_t}}, \mathbf{g}_{T_t}^{(n)}\right)\right\|_2^2 \quad \textbf{(8)}$$

These two subproblems continue jointly updating both $\mathbf{g}_{\mathrm{T_t}}$ and $\mathbf{x}_{\mathrm{T_t}}$ in a sequential fashion until the error between successive estimates become negligible, and then the unconstrained and constrained KF process is repeated updating the time set, T_{t+1} until the time set reaches to the final time frame.

Results: A cardiac cine data ($N_{pe} \times N_{fe} \times N_t$: $232 \times 256 \times 24$) is simulated for a 8-channel body coil array. To emulate undersampling, Poisson-disk random undersampling was used with 12 center k-space lines at the Nyquist sampling rate. To evaluate the performance of the proposed algorithm, we compare the results of our algorithm with those of k-t Sparse SENSE at acceleration factor of 5 and 8. In Fig.2 and3, the proposed method effectively suppresses aliasing artifacts and noises preserving spatial and temporal resolution while k-t Sparse SENSE exhibits considerable loss of myocardium edge sharpness at high acceleration factor.

Conclusion: By applying low rank and sparsity constrains to constrained KF estimation, the proposed method updates the kernel and image sequence jointly to improve the reconstruction quality of cardiac cine imaging. The results of the proposed method show superior performance than the existing method.

References: [1] Tsao et al, MRM, 2003;50:1031, [2] Huang et al., MRM, 2005;54:1172, [3] Otazo et al., MRM, 2010;64:767, [4] Lustig et al., ISMRM, 2006;2420.

Acknowledgements: Mid-career Researcher Program (2011-0016116), NRF KOREA, MEST.

a

$$\underbrace{\begin{pmatrix} \mathbf{y}_{\gamma,t}^{\mathrm{Nyq}} \\ \mathbf{y}_{\gamma,t+1}^{\mathrm{Nyq}} \\ \vdots \\ \mathbf{y}_{\gamma,t+\tau-1}^{\mathrm{Nyq}} \end{pmatrix}}_{\mathbf{y}_{\gamma,T_t}^{\mathrm{Nyq}}} = \underbrace{\begin{pmatrix} \mathbf{S}_t^{\mathrm{Nyq}} & & & \\ & \mathbf{S}_{t+1}^{\mathrm{Nyq}} & & \\ & & \ddots & \\ & & & \mathbf{S}_{t+\tau-1}^{\mathrm{Nyq}} \end{pmatrix}}_{\tilde{\mathbf{S}}_{\gamma,T_t}^{\mathrm{Nyq}}} \underbrace{\begin{pmatrix} \mathbf{g}_{\gamma,t} \\ \mathbf{g}_{\gamma,t+1} \\ \vdots \\ \mathbf{g}_{\gamma,t+\tau-1} \end{pmatrix}}_{\mathbf{g}_{\gamma,T_t}}$$

$$\mathbf{g}_{\mathrm{T_t}}^{-} = \mathbf{g}_{\mathrm{T_{t-1}}}$$

$$\mathbf{P}_{\mathrm{T_t}}^{-} = \mathbf{P}_{\mathrm{T_{t-1}}}^{+} + \mathbf{Q}$$

$$\hat{\mathbf{g}}_{\mathrm{T_t}} = \mathbf{g}_{\mathrm{T_t}}^{-} + \mathbf{K}_{\mathrm{T_t}}\left(\mathbf{y}_{\mathrm{T_t}}^{\mathrm{Nyq}} - \tilde{\mathbf{S}}_{\mathrm{T_t}}^{\mathrm{Nyq}}\mathbf{g}_{\mathrm{T_t}}^{-}\right)$$

$$\mathbf{K}_{\mathrm{T_t}} = \mathbf{P}_{\mathrm{T_t}}^{-}\left(\tilde{\mathbf{S}}_{\mathrm{T_t}}^{\mathrm{Nyq}}\right)^H\left(\tilde{\mathbf{S}}_{\mathrm{T_t}}^{\mathrm{Nyq}}\mathbf{P}_{\mathrm{T_t}}^{-}\left(\tilde{\mathbf{S}}_{\mathrm{T_t}}^{\mathrm{Nyq}}\right)^H + \mathbf{R}\right)^{-1}$$

b $$\mathbf{P}_{\mathrm{T_t}}^{+} = \left(I - \mathbf{K}_{\mathrm{T_t}}\tilde{\mathbf{S}}_{\mathrm{T_t}}^{\mathrm{Nyq}}\right)\mathbf{P}_{\mathrm{T_t}}^{-}$$

Figure. 1. *Illustration of (a) the block matrices of semi-batch type approach and (b) the Kalman filter estimation*

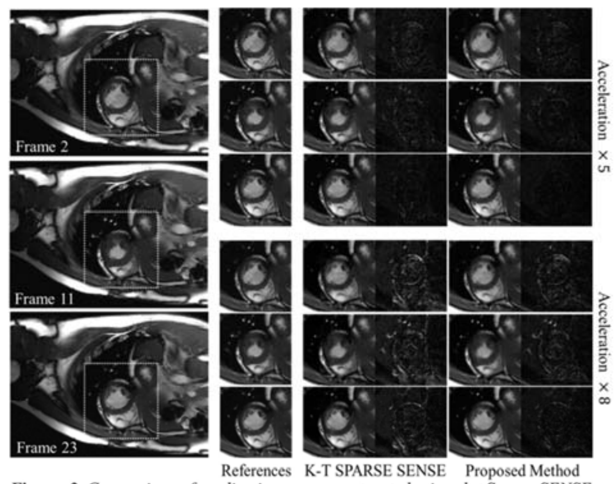

Figure. 2. *Comparison of cardiac images reconstructed using: k-t Sparse SENSE and the proposed method at acceleration factor of 5 and 8.*

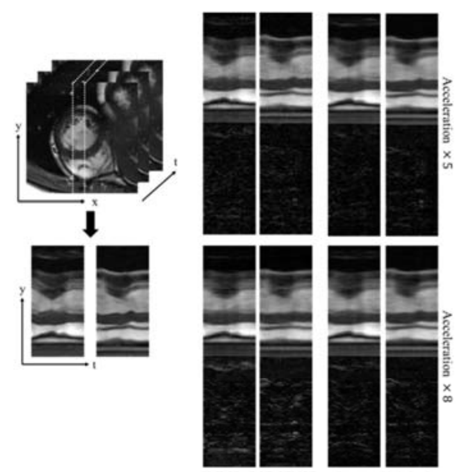

Figure. 3. *Comparison of temporal resolution reconstructed using: k-t Sparse SENSE and the proposed method at acceleration factor of 5 and 8.*