Incoherence Parameter Analysis for Optimized Compressed Sensing with Nonlinear Encoding Gradients

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Target Audience: The compressed sensing and non-linear gradient encoding communities. **Purpose**: The incoherence parameter as defined below determines compressed sensing convergence (CS).[1,2] The incoherence parameter is visualized through pairwise incoherence maps and minimized to improve CS performance. O-space imaging[3], a nonlinear magnetic gradient field encoding parallel imaging method, is shown to produce levels of incoherence that exceed levels in linear encoding methods for a 2D slice. The current work proposes optimizing the incoherence parameter using nonlinear gradients to improve the convergence performance for CS.

Methods: To ensure the restricted null space property, which similar to the restricted isometry property is sufficient to guarantee convergence in CS, the pairwise incoherence parameter must satisfy: $\delta \coloneqq max\left|\frac{\langle(\Phi\Psi^{-1})_i|(\Phi\Psi^{-1})_j\rangle}{N}\right| \leq \frac{1}{3k} \forall i \neq j$, where Ψ is the sparsifying transform, Φ is the sensing basis (encoding matrix), indices refers to a single row in $\Phi\Psi^{-1}$, N is the number of pixels in the image, and the sparsity cardinality k specifies the largest subset of vectors in the sparse domain that will be exactly recovered.[1,2,4] The pulse sequence, gradient fields, and receiver array are shown in fig. 1. The Z^2 coefficients are iterated 20 times within a 30% range of nominal to find a best "randomized" set to produce maximum incoherence.[5] The reconstruction used is the Kaczmarz iterative algebraic projection reconstruction (ART) algorithm, which is a type of projection on to convex sets algorithm that converges to the minimum norm solution.[6] The sparsity-promoting convex optimization uses the Daubechies wavelets and total variation. The argument of δ is measured at every point in the grid (fig. 1) and a sample value is given for single and double detectors.

Results: The incoherence parameter was successfully reduced by up to 51.6% at points moving from single to double receivers for pseudo-random O-space (see tab. 1 showing values for arg{δ} and fig.2) and the image MSE was up to 21.9% less when optimized (fig. 3). **References**: [1]Donoho, D. et. al. IEEE Info Thry 2006; 52(12):5406. [2]Cohen, A. et. al. J. AMS 2009; 22(1):211. [3]Stockmann, J. et. al. MRM 2010. 64: 447. [4]Donoho, D., Huo XM. IEEE Info. Thry. 2001;47(7):2845. [5]Tam, L.K. et. al. ISMRM 2013, p2611. [6]Herman G.T. et. al. J. Theo. Biol. 42:1. [6]Liang et al. MRM 2009. 62: p. 1574-1584.

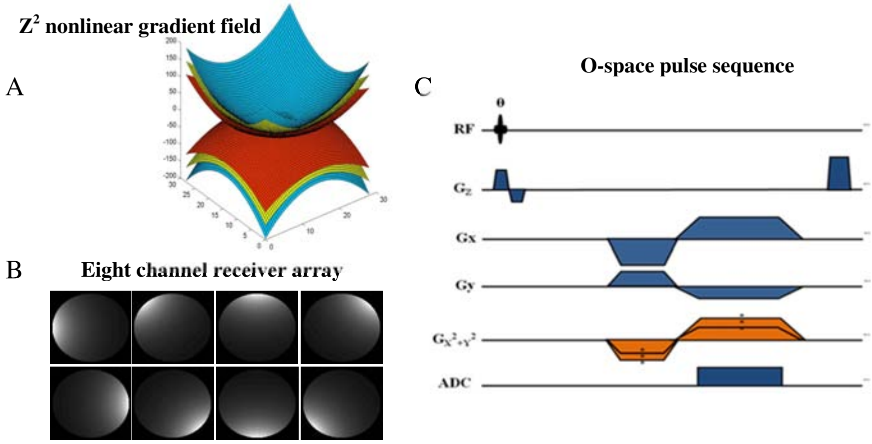

Fig. 1. (A) Isocontours of the nonlinear Z^2 ($= z^2 - 1/2(x^2+y^2)$ spatial variation) magnetic encoding field. (B) Receiver sensitivity profiles of an 8 channel receiver array used in parallel imaging. (C) O-space is a gradient echo sequence, with 3 encoding fields, the two linear fields and a nonlinear Z^2 field. The Z^2 strength may be perturbed to optimize the incoherence parameter[5], which determines the number of sparsity vectors exactly recovered in CS.

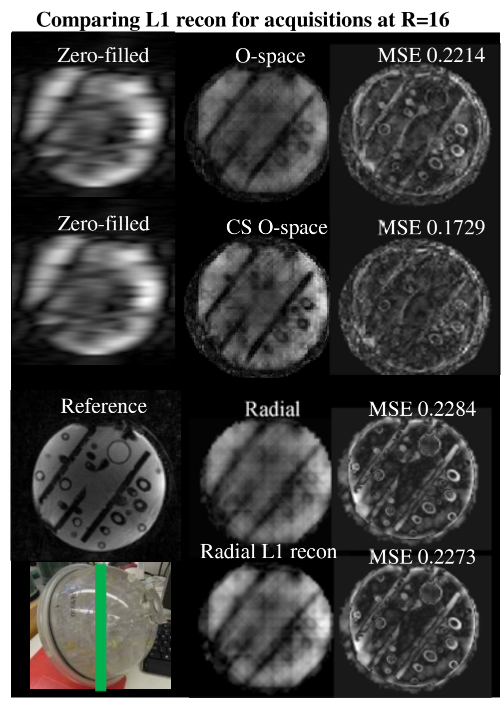

Fig. 3. 128x128 images of a contrast phantom at R=16 and difference images comparing algebraic reconstruction for O-space, Radial, and their CS optimized counterparts. Experiments were on a 3T Siemens MAGNETOM Trio TIM. SENSE is intractable due to noise amplification at R=16. MSE is reduced when the incoherence parameter is reduced via perturbation of the nonlinear gradient.

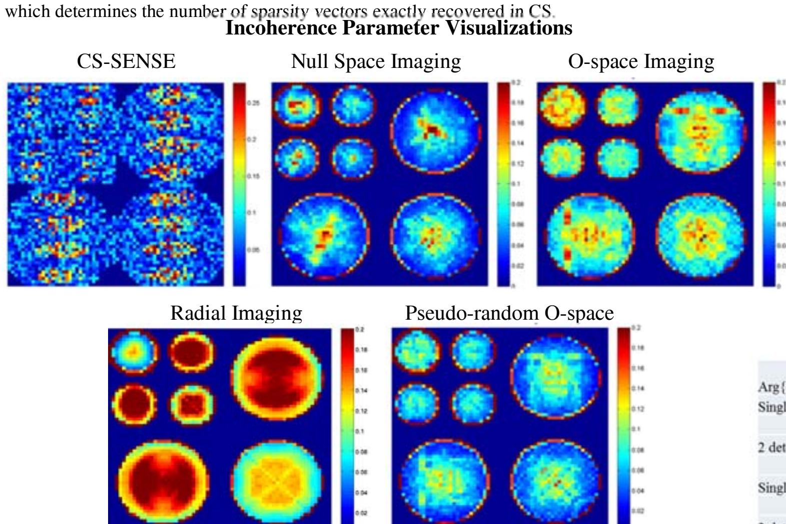

Fig. 2. Incoherence parameter maps (arg{δ}) visualize the pairwise incoherence for CS-SENSE[6], NSI, O-space, and pseudo-random CP O-space methods by computing the pairwise incoherence at every point in the grid. A lower value (blue) indicates better incoherence and leads to recovery of more sparse vectors. CS-SENSE shows SENSE noise amplification patterns that undesirably increase the incoherence parameter. Pseudo-random O-space has the lowest SPR values overall.

Table 1. Arg{δ} values at (9,27) (lower is better)

Arg{δ}	Linear	Linear (randomized along readout)	Phase encoding	O=space (gradient strength randomized)
Single detector, 1/4 data	0.3508	0.3359	0.3395	0.1326
2 detectors, 1/4 data	0.2237	0.2149		0.0684
Single detector, 1/8 data	0.8571	0.8469	0.633	0.2213
2 detectors, 1/8 data	0.8168	0.8436		0.1097