Data-driven Cartesian sampling design for Compressed Sensing MRI

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Introduction: Random sampling is one of the key components of Compressed Sensing (CS) in MRI. Lustig et al. have shown that variable density (VD) sampling is often preferable to uniform random sampling due to the concentration of energy in the center of k-space [1]. We recognize this as a data dependency, i.e. choosing an optimal sampling density is difficult without knowledge of the kind of data to be reconstructed. We consider it possible to design sampling patterns for CS using a data-driven approach, where fully sampled reference scans are used as a training set. Instead of drawing samples randomly, we choose to optimize a sampling pattern such that CS reconstruction performs well on this training set. As long as a sufficient number of representative reference scans are used, the resulting sampling pattern can be used for similar scans outside the training set. Here, we propose a simple iterative scheme to heuristically optimize a Cartesian sampling pattern for a given training set. The performance of this method is evaluated against the commonly used VD sampling pattern.

Methods: Given a training set, the sampling pattern can be iteratively optimized. The method starts with a sampling pattern in which only the center coefficient of k-space is sampled. Then, the following steps are iterated I times (where I is sufficiently large, we use $I = 100$):

1. The training set is reconstructed retrospectively using CS with the current sampling pattern
2. The reconstruction error of unmeasured points in k-space is quantified over the entire training set by mean square error (MSE)
3. The M points with the highest MSE are added to the current sampling pattern, with M such that after the addition we have $round(i/I \cdot N/A)$ points (with i = current iteration nr., N = total nr. of points (without undersampling) and A = acceleration factor).

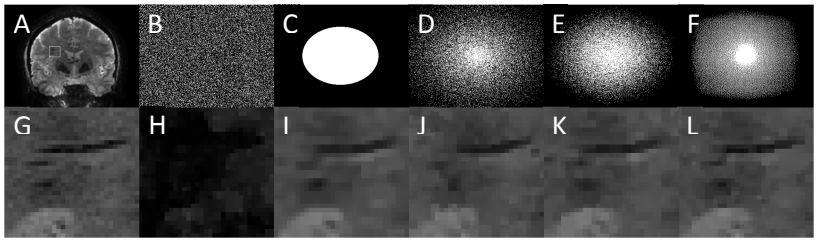

Figure 1. *Fully sampled slice with cutout (A/G). Sampling patterns and cutout from CS reconstruction for sampling methods: random (B/H), low resolution (C/I), VD (Lustig) (D/J), VD (Optimized) (E/K) and iterative (F/L).*

For comparison, VD sampling was adapted to be optimized on the training set. For each sampling density p, a probability density function $\max(1 - x, 0)^p$ is generated (where x is the distance to the centre of k-space, p was varied between 1 and 4 with 0.25 increments), and M random patterns are drawn ($M = 25$). The training set is reconstructed using CS for each of the generated sampling patterns. The sampling pattern with the lowest Normalized Root Mean Squared Error (NRMSE) is then chosen as the optimal pattern. Additionally, we used VD sampling with optimization of incoherence as implemented by Lustig [1]. This method yields only one sampling pattern per sampling density (varied between 1 and 4 with 0.25 increments). The optimal density (as per NRMSE) was chosen by reconstructing the training set with CS using the generated sampling patterns. Finally, we compare the results of all the obtained sampling patterns with one single uniform random sampling pattern and low resolution sampling with CS reconstruction.

Experimental setup: Previously acquired 3D T2*-weighted brain scans (matrix 480x381x300, TR/TE = 70/27, EPI factor = 13x, SENSE (LR) = 2.3x) of seven subjects were used, acquired at 7T (Philips Healthcare, Cleveland, USA) [2]. From each 3D scan we selected 15 coronal slices, equally spaced from front to back. All undersampling is done in the phase encoding plane (i.e. the coronal plane), to realistically allow 2D undersampling. The dataset was split for 7-fold cross validation, such that the slices from one subject are reconstructed with CS using a sampling pattern trained on all selected slices from all other subjects. A CS acceleration factor of 4x was chosen. CS reconstruction is performed with both Wavelet and Total Variation regularization, using a non-linear Conjugate Gradient optimizer. The regularization weights were optimized with parameter sweeps on a single mid-coronal slice from the first subject ($\lambda_{TV} = 2.4 \cdot 10^{-3}$, $\lambda_W = 1.6 \cdot 10^{-3}$).

Evaluation: The CS reconstructions for each method were evaluated using both NRMSE and the mean Structural SIMilarity (mSSIM) index [3]. The mSSIM index was only evaluated on parts of the image where the magnitude is higher than 1% of the maximum magnitude.

Results: The main results for the evaluated methods are shown in Table 1. The error metrics are reported as a mean over all seven cross validations. Figure 1 shows the resulting sampling patterns, as well as CS reconstructions for a single slice.

Method	Mean NRMSE	Mean mSSIM	Training time
Random	11.43%	0.498	n/a
Low resolution	4.39%	0.818	n/a
VD (Lustig)	3.60%	0.812	1:41h
VD (Optimized)	3.30%	0.843	56:47h
Iterative	**3.10%**	**0.851**	18:15h

Table 1. *Evaluation results for different sampling methods*

Discussion: For the analyzed dataset, there is a definite benefit in optimizing a VD sampling pattern, since both NRMSE and mSSIM clearly improve compared to Lustig's implementation. On top of that, there is a further improvement when using the proposed iterative design method. Because cross-validation was used to obtain the results it can be subsumed that the results will generalize to new scans, given that the new scan is of the same type and follows the same protocol as the reference scans. Both the optimized VD and iteratively designed sampling pattern slightly lower the effective resolution by not sampling the highest frequencies (as seen in Figure 1E/1F). We consider this a limitation of the requested acceleration factor.

One of the main advantages of the proposed method is that it explicitly uses the reconstruction method in the optimization process. Therefore, it theoretically generalizes to any reconstruction method that allows Cartesian undersampling in any number of dimensions. This would, for example, enable optimization of sampling patterns for a combination of CS and parallel imaging, which is not directly possible with the other methods. Our method furthermore facilitates the interesting possibility of automatically determining an optimal acceleration factor, by iterating until a given error value is reached. A drawback of the current implementation of the method is that fully sampled reference scans are required, which may be hard to obtain.

Conclusion: We have shown that even with a simple method sampling patterns for CS can be improved using a data-driven approach. The proposed iterative method outperforms other methods according to both the NRMSE and mSSIM metrics. On top of that the method can theoretically be applied to many different types of scans and can potentially include parallel imaging.

References: [1] Lustig et al., MRM, 2007; 58(6):1182-95 [2] Zwanenburg et al., Neuroimage, 2011; 15;56(4):1902-7 [3] Wang et al., IEEE TIP, 2004; 600-612