

# Sampling pattern design for 2D compressed sensing using a multilevel variable-density spiral trajectory

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**Purpose:** An important design element in compressed sensing (CS) MRI is the choice of sampling pattern. Variable-density random k-space sampling patterns are particularly effective for 3D Cartesian sampling, where there is flexibility in choosing ky and kz phase encoding positions [1, 2]. The design of 2D sampling patterns for high acceleration is more challenging. Adcock et al. recently developed a framework that describes MRI as both asymptotically sparse and asymptotically incoherent, and using this framework they designed an optimal multilevel random subsampling scheme [3]. The purpose of this work is to apply this subsampling scheme for 2D CS MRI with high acceleration using a variable-density spiral trajectory.

**Methods:** Radial trajectories have shown promising results [3] using undersampling of radial spokes, resulting in a linear sampling probability function as a function of k-space radius. A similar linear undersampling scheme has been used for spiral trajectories [5]. However, linear undersampling is suboptimal [3]. Variable-density spiral trajectories permit complete flexibility in choosing sampling density as a function of radius and thus are ideal for implementing multilevel random subsampling.

The multilevel strategy separates k-space into N concentric circular regions. Each region follows the following sampling probability or density function:

$$r_k = \begin{cases} m & k = 0 \\ k \frac{(1-m)}{N-1} & k = 1, \dots, N-1 \end{cases} \quad p_k = \exp\left(-\left(\frac{bk}{N}\right)^a\right)$$

$p_k$  is the sampling probability in the k-space region with radius  $r_k$ . A spiral trajectory can be expressed as

$$k = Ate^{i\omega t}$$

If we let  $\omega$  follow the sampling density  $p_k$ , the density of the resulting spiral trajectory as a function of radius matches the ideal random subsampling density, as shown in Fig. 1.

To demonstrate the performance of the proposed spiral design, a numerical phantom [6] was used to simulate a 2D spiral acquisition. Multiple interleaves were chosen to achieve the target resolution and each interleaf was rotated by a random angle to provide incoherent sampling. A constant-density spiral trajectory sampled at the Nyquist rate was used as a reference and reconstructed using a conjugate gradient method. Experimental data was collected on a Siemens 3T Trio scanner with a single-channel coil. Data from a resolution phantom was acquired in Cartesian k-space and then pseudo-undersampled to a gridded spiral trajectory. Finally, the proposed spiral trajectory was directly implemented and its performance was compared to a constant-density spiral trajectory. All undersampled data was reconstructed by a general forward-backward splitting method to exploit sparsity in total variation (TV) and wavelet domains.

**Results:** Fig. 2 shows results from multilevel density spiral with CS reconstruction. In the simulation (a), the proposed spiral trajectory achieved an acceleration rate of 5, without noticeable aliasing. The gridded spiral experiment (b) achieved similar results with acceleration rate of 8. In the experiment with the proposed trajectory (c), the non-uniform Fourier transform amplified the trajectory error in the iterative reconstruction, somewhat limiting its performance. (c2) shows the result with an acceleration rate of 2.

**Conclusion:** The flexibility of variable-density spiral trajectories gives them unique advantages for optimal sampling pattern design for 2D compressed sensing. This study demonstrates that these trajectories can be used to implement a theoretically optimal sampling pattern, and that this pattern yields high acceleration rates for a single-channel 2D acquisition with pseudo undersampling. Further work is needed to optimize this trajectory experimentally and to determine the limits of 2D acceleration.

**Reference:** [1]. Lustig et al. MRM 2007;58:1182. [2] Zhang et al. JMRI 2013. [3]. Adcock et al. arXiv:1302.0561. [4]. Qing et al. ISMRM 2011;546. [5] Lustig et al. ISMRM 2005;685. [6]. Guerquin-Kern et al. IEEE Trans Med Imaging. 2012;626-636.

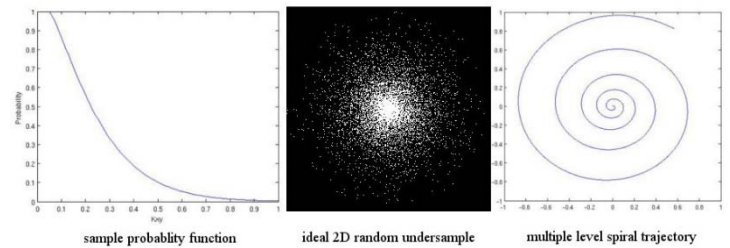


Fig. 1. Multilevel spiral sampling density design

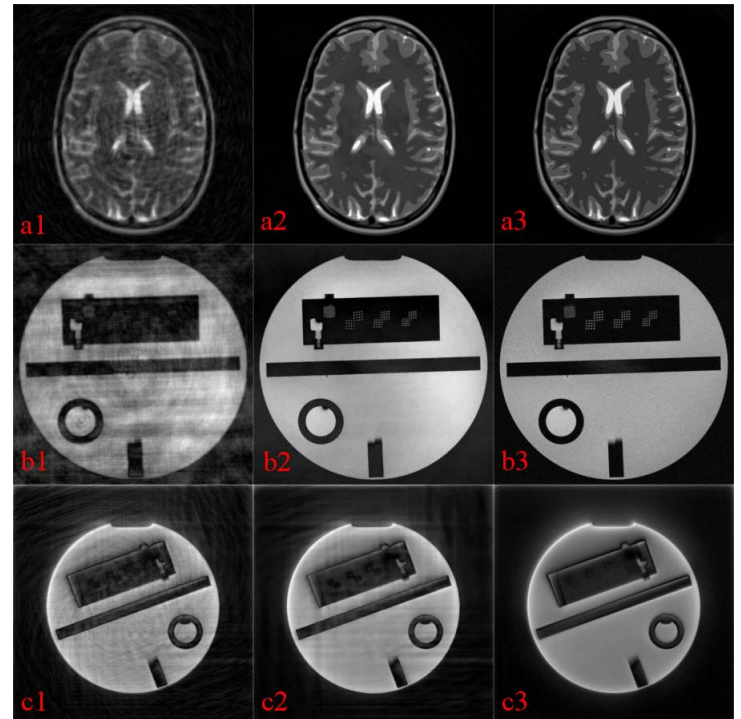


Fig. 2. Numerical phantom simulation results (a) show that the proposed multilevel spiral with 11 interleaves and 10ms readout contains aliasing artifacts using traditional reconstruction (a1). The compressed sensing reconstruction (a2) has negligible artifacts compared to the fully sampled reconstruction (a3). In experiment (b), fully sampled Cartesian data (b3) was pseudo undersampled (12.4%) to a gridded spiral trajectory. Compressed sensing (b2) eliminated most artifacts apparent in the direct reconstruction (b1) and kept the structural details. Experiment (c) was a multilevel spiral experiment with 47 interleaves and 6 ms readout with a resolution of 0.5mm. The direct reconstruction is shown in c1 and the compressed sensing result in c2. The fully-sampled acquisition required 95 interleaves (c3).