

Automated Design of Time-Optimal SWIRLS-like trajectory Additionally Optimized for Sampling Uniformity

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Target audience: Those who intend to design or evaluate 3D non-Cartesian trajectories.

Purpose: Shells is a 3D non-Cartesian trajectory that samples helical spirals on the surfaces of concentric spheres. It has been shown to have many desirable properties such as time-optimal acquisition of the center of k-space, easily-varied sampling density with radius, and efficient overall coverage of k-space.^{1,2} SWIRLS is an improved version of shells that mitigates oversampling of the poles and is more tolerant of gradient slew limits.³ It accomplishes this by transitioning the trajectory to follow a line of longitude as it approaches the poles of the sphere. Traditionally, a default SWIRLS trajectory design requires manual intervention to select appropriate transition points for the trajectory on each individual shell. This must be done for each spatial resolution, FOV, and scanner hardware limitations the operator wishes to use. The purpose of this work is to demonstrate a new, flexible trajectory design inspired by SWIRLS that is not only automated given any set of input parameters, such as gradient performance, FOV, resolution, etc., but also produces an improved trajectory which is optimized for 3D sampling density uniformity both across the shells and on each individual shell. The proposed trajectory is referred to as AutoSwirl. **Methods:** The AutoSwirl trajectory is similar to SWIRLS. However, instead of a segmented model, a single, continuous trajectory is defined:

$$k_x(a, u) = k_r \cos\left(\frac{\pi}{2}u\right) \sin\left(R \frac{\pi}{2}S(a, u)\right) \quad k_y(a, u) = k_r \cos\left(\frac{\pi}{2}u\right) \cos\left(R \frac{\pi}{2}S(a, u)\right) \quad k_z(u) = k_r \sin\left(\frac{\pi}{2}u\right) \quad S(a, u) = \left[(1 + e^{-au})^{-1} - \frac{1}{2}\right] \left[(1 + e^{-a})^{-1} - \frac{1}{2}\right]^{-1} \quad (1)$$

where k_r is the k-space radius of the shell, $u \in [-1, 1]$, R controls the number of rotations an interleave completes from pole-to-pole, and a determines the slope of the sigmoid $S(a, u)$. $S(a, u)$ is defined as to always satisfy $S(a, u) \in [-1, 1]$. The sigmoid shape parameter a $\{a \in \mathbb{R}: a > 0\}$ is chosen by minimizing the following cost functional: $J(a) = \left\| \text{area}\{CH\{L(a, :)\}\} - \text{mean}\{\text{area}\{CH\{L(a, :)\}\}\} \right\|_1$ (2) where $L(a, :)$ returns a vector of points sampled from the line described in (1) which

correspond to the time-optimal traversal of (1), subject to a specified maximum gradient amplitude and slew rate. The operation $CH\{\}$ computes the convex hull of a completed shell, while $\text{area}\{\}$ returns a vector containing the areas of the triangles from the convex hull.

Thus, the optimized value of a is one that generates a trajectory whose points are most uniformly distributed on the surface of the sphere. The time-optimal gradient design operator, $L(\cdot, :)$, is implemented via the algorithm described in⁴, using publicly-available code⁵. R is then selected through iteration so the number of samples on a single interleaf equals the number of samples desired for a single TR. The number of interleaves for a given sphere is chosen following the guidelines for a fully sampled shells trajectory in⁶. The minimum of $J(a)$ was found by a brute force line search over 40 points of $a \in [1, 4]$ linearly spaced. a was observed to span the range of solutions for the parameters used. The sampling of 40 points in the search space was determined to be sufficient with little to no accuracy being gained with finer sampling. The algorithm was implemented in MATLAB[®]. Computation took 8.6 min using 12 workers of a Linux system with dual Intel[®] Xeon[®] E5-2670 CPUs. Comparison scans using both the new and old techniques were acquired on a 1.5T scanner (GE Signa, v16.0, Milwaukee, WI), both with a TR of 7.4 ms, TE of 3.1 ms, flip angle of 10°, FOV of 24 cm², an isotropic resolution of 1.0 mm³, 512 points sampled per interleave with a ± 62.5 kHz sampling bandwidth, and the same number of interleaves for a given shell. The total scan time was 2 minutes and 4 seconds for both methods. A resolution phantom was scanned with a single channel T/R head coil and a volunteer was scanned under an IRB-approved protocol, also with a single channel T/R head coil. Reconstructions were done using a NUFFT⁷ based gridding procedure with 1.25x oversampling and density compensation via Pipe and Menon⁸. The volunteer scan also has B₀ off-resonance correction applied. Bulk average gradient delay was also applied during the reconstruction.

Results: Figure 1 shows the improvement in sampling density uniformity according to the metric described in equation 2. The values shown in Figure 1 have been normalized by the length of the vector returned by $\text{area}\{\}$ to better illustrate the relative uniformity over different shells. A lower value using the metric in (2) means the areas among points sampled on that shell are more uniform. The k-space points used to generate Figure 1 match the acquisition experiment described in the methods section. Of note is the large difference in sampling uniformity for the lower numbered shells. Figure 2 shows examples of a phantom and volunteer study comparing the default SWIRLS trajectory to the AutoSwirl trajectory. The default SWIRLS trajectory can exhibit substantial artifacts, which are most visible in planes that include the z-axis, as can be seen in (b) and (c). The AutoSwirl yields improved image quality in even the xy-plane, as seen in (a), and exhibits fewer artifacts when viewed in other planes; thus demonstrating a benefit of a trajectory which has been optimized for uniform 3D sampling density.

Discussion: In practice, the default SWIRLS trajectory would be corrected by acquiring additional interleaves to remove the artifacts seen in Figure 2. Because of the more uniform sampling in the AutoSwirl trajectory these extra interleaves are not necessary, which improves its efficiency.

Conclusion: We proposed a new method for designing a shells-like trajectory which samples k-space with improved uniformity relative to previous designs. As demonstrated, this modification leads to improved image quality and reduced artifacts compared to previous SWIRLS designs. In addition, the algorithm described above will generate an AutoSwirl trajectory automatically for any given set of parameters of FOV, resolution, readout length, or scanner hardware limitations, allowing the AutoSwirl trajectory to be more easily implemented in a variety of clinical and research situations.

References: [1] Y. Shu et al. MRI 24:739-749,2006; [2] Y. Shu, et al., MRM, 56:553-562, 2006; [3] Y. Shu et al., ISMRM 2009:4536 [4] M. Lustig et al., IEEE Trans Med Imag, 27:866-873, 2008; [5] M. Lustig, www.eecs-berkeley.edu/~mlustig/Software.html [6] Y. Shu et al., MRM 56:553-562, 2006; [7] J. Fessler, IEEE Trans Sig Proc, 51:560-574, 2003; [8] J.G. Pipe, P. Menon, MRM, 41:179-186,1999

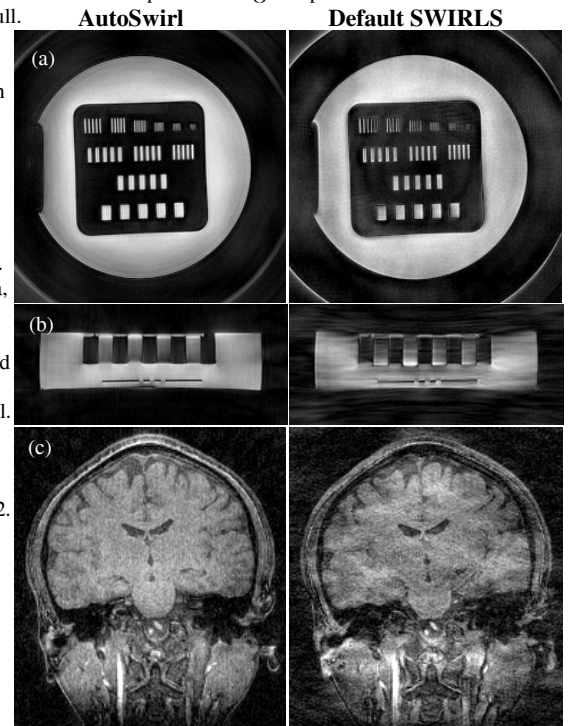
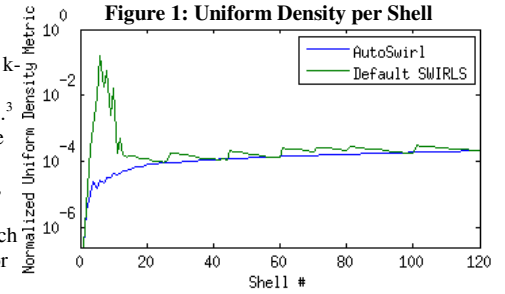


Figure 2: Orthogonal slices through the center of a resolution phantom and coronal slice from an example volunteer scan using the default SWIRLS trajectory (right) without correction and the proposed AutoSwirl trajectory (left). Slices in (a) are in the xy-plane, slice in (b) and (c) are in the xz-plane. Row (b) has been vertically cropped to conserve space.