

# Amplitude-modulated continuous wave excitation

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**TARGET AUDIENCE** Researchers with an interest in continuous wave RF excitation.

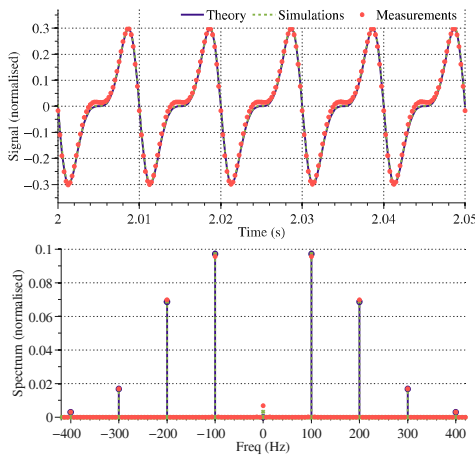
**PURPOSE** There is renewed interest in continuous wave (CW) excitation for imaging samples with very short  $T_2$  times<sup>1</sup>. CW techniques such as stochastic MRI<sup>2</sup> and SWIFT<sup>1</sup> have assumed that the spin system can be treated in a linear time-invariant framework, a valid assumption under certain conditions<sup>3</sup>. Motivated by work in optics<sup>4</sup>, it is our aim here to investigate the nonlinear interactions revealed by the use of amplitude modulated CW excitation. We demonstrate novel magnetization behavior not previously observed in magnetic resonance systems.

**THEORY** The spin system is excited with an RF field rotating at the Larmor frequency with an amplitude envelope given by  $\gamma B_1^e(t) = \omega_1(1 + \alpha \cos(\omega_m t))$  where  $\omega_1$  is the average Rabi frequency,  $\alpha$  is the modulation factor and  $\omega_m$  the modulation frequency. A solution to the Bloch equations was derived for  $\omega_1 = \omega_m$  using averaging techniques<sup>5</sup>. The transverse components of the steady-state magnetization are  $m_x(t) = 0$  and

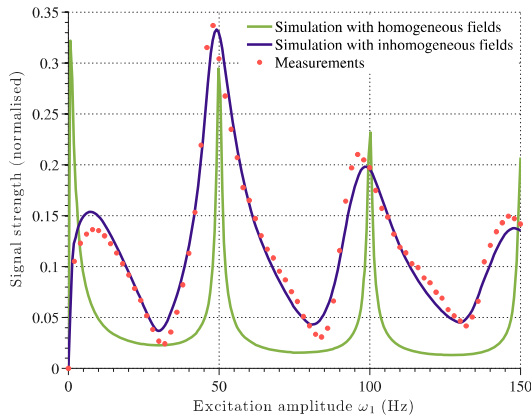
$$m_y(t) = A \sin(\omega_m t + \alpha \cos(\omega_m t)) \quad \text{where} \quad A = \frac{-2R_1 J_1(\alpha)}{R_1 + R_2 + J_2(2\alpha)(R_1 - R_2)}, \quad (1)$$

$R_1$  and  $R_2$  are longitudinal and transverse relaxation rates, respectively, and  $J_k$  is the  $k^{\text{th}}$  order Bessel function of the first kind. Eq. (1) can be written as a Fourier series to emphasize the frequency components contributing to the observable magnetization.

$$m_y(t) = A \sum_{k=1}^{\infty} a_k \sin(k\omega_m t) \quad \text{where} \quad a_k = J_{k-1}(\alpha) - J_{k+1}(\alpha) \quad (2)$$



**Fig. 1** (a) Measured (circles), theoretical (solid) and simulated (dashed)  $y$ -component of the magnetization and (b) corresponding frequency components demonstrating higher-order harmonics.



**Fig. 2** Amplitude of the steady state magnetization simulated with homogenous fields (green), inhomogeneous fields (blue) compared to measurements (circles). A large signal is observed when  $\omega_1$  is a multiple of  $\omega_m$ .

**METHODS** Experiments were performed with a spherical phantom of Gd-doped water ( $T_1=342\text{ms}$ ,  $T_2=139\text{ms}$ ) on a 4.7T Bruker BioSpec small bore MRI scanner. The steady state magnetization was measured using a series of FID experiments as follows. The sample was excited with an amplitude-modulated envelope for an initial duration of 2s. Immediately after the excitation an FID was acquired and the first point was extracted to reflect the state of the magnetization at that time. This process was repeated for 600 FIDs each following an excitation for 250 $\mu\text{s}$  longer than the previous excitation to accurately track the magnetization for 150ms. This proof-of-concept technique was implemented without additional hardware. A spectrum was obtained by a DFT of the signal waveform. The first experiment excited with  $\alpha = 1$  and  $\omega_1 = \omega_m = 100\text{Hz}$ . A second experiment excited with amplitudes defined by  $\omega_1$  ranging from 0 and 150Hz in 2Hz increments for a fixed  $\alpha = 1$  and  $\omega_m = 50\text{Hz}$ . FIDs were acquired with a dwell time of 100 $\mu\text{s}$ . **Field mapping:** The distribution of  $B_0$  off-resonance was measured using a field-mapping sequence (FOV=65mm isotropic, 128x128x128 matrix) developed for shimming applications (MAPSHIM, Bruker Biospin). The distribution of  $\omega_1$  across the FOV was measured with a  $B_1$  mapping sequence<sup>6</sup> (TR=1s, FOV=80mm isotropic, 256x256 matrix, 2mm slices). Distributions were extracted using a histogram of non-background voxels. **Simulations:** The magnetization was simulated by integrating numerical solutions to the Bloch equation over the measured  $B_0$  and  $B_1$  distributions. All simulations and data processing were conducted using MATLAB.

**RESULTS** Fig. 1a illustrates the  $y$ -component of the magnetization under the influence of amplitude modulated CW excitation. The measurements (circles) are in excellent agreement with the theory in Eq. (1) (solid line) and simulations (dashed line). The magnetization rotates about the  $x$ -axis at a variable speed dependent on the Rabi frequency. The corresponding spectrum (Fig. 1b circles) agrees with theoretical expressions in Eq. (2) (Fig. 1b solid line). Fig. 2 illustrates the amplitude of the steady state signal for varying RF amplitudes. The result demonstrates that it is necessary to include the field inhomogeneities in simulations (solid line) to properly explain the measurements (circles). Importantly, the steady state signal is large when the RF amplitude is matched to a multiple of the modulation frequency (i.e. 50Hz, 100Hz, 150Hz, etc.), establishing a secondary resonance condition called ‘Rabi resonance’<sup>4</sup>.

**CONCLUSION** These proof-of-concept experiments demonstrate, for the first time, the response of the magnetization to CW excitation with a modulated amplitude envelope. A large steady state signal with higher-order harmonics is observed when the Rabi resonance condition is satisfied. We are currently developing new methods in MRI and spectroscopy that exploit these fundamental phenomena<sup>7</sup>.

**REFERENCES** <sup>1</sup>Idiyatullin et al. 2012 J Magn Reson 220:26–31 <sup>2</sup>Nilgens et al. 1996 Mag Res Imag 14:857–861 <sup>3</sup>Ernst et al. 1987 Oxford Uni Press <sup>4</sup>Cappeller & Müller 1985 Annal Phys 497:250–264 <sup>5</sup>Tahayori et al. 2008 IEEE CDC 4121–4126 <sup>6</sup>Wang. MRM. 2005; 53:408–417 <sup>7</sup>\_\_\_\_ ISMRM 2014