# AN EASILY CONTROLLABLE SPREAD SPECTRUM USING CHIRP RADIO FREQUENCY PULSE AND ITS APPLICATION IN COMPRESSED SENSING MRI

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**Introduction:** To accelerate the MRI, compressed sensing (CS) [1] suggests performing randomly undersampling and reconstructing the image in a sparse transform domain. Random sampling is necessary since it will reduce the coherence between the encoding matrix and the sparsity bases[2]. Recently, the spread spectrum is introduced to reduce the coherence thus improve the reconstruction [3]. However, spread spectrum is achieved via a second order shim coil which limits the modulation intensity and is not convenient to be operated. In this work, we introduce a chirp radio frequency (RF) pulses to easily control the spread intensity by choosing a proper bandwidth of this pulses and apply it in CS MRI.

**Methods:** The designed sequence (Fig. 1) is a variant of the conventional multi-scan spin-echo sequence [4, 5]. The chirp pulse has a linear frequency modulation as $\omega_{RF}(t_{\text{enco}}) = O_0 + Rt_{\text{enco}}$ where $R$ and $O_0$ are the chirp rate and the initial frequency of the pulse. Bandwidth of this pulse is $\Delta O = RT_{\text{enco}}$. In CS, the phase variation generated by decoding gradient is $\varphi_{\text{deco}}(t) = -n\gamma\Delta g_{\text{deco}} t_{\text{deco}} y$ where $\Delta g_{\text{deco}}$ is the incremental magnitude of the decoding gradient between two successive scans, $t_{\text{deco}}$ is the duration of the gradient, and $n$ is the index of the scanning cycle chosen from. With series of derivation [6], the acquired signal for the sequence is

$$s(m) = \int_0^{L_Y} \rho(y) e^{i\phi(y,m)} dy = \int_0^{L_Y} \rho(y) e^{-ik_m y} \phi(y) dy \qquad (1)$$

where $k_m = -\frac{m}{N}\frac{\Delta O \cdot T_{\text{enco}}}{L_Y}$ and $\phi(y) = e^{-i\left[\frac{\Delta O \cdot T_{\text{enco}}}{2L_Y^2} y^2 + \frac{1}{8}\Delta O \cdot T_{\text{enco}} + \frac{\pi}{2}\right]}$. A discrete form of Eq. (1) is

$$\mathbf{s} = \mathbf{UF}\tilde{\mathbf{\Phi}}\boldsymbol{\rho} \qquad (2)$$

where $\tilde{\mathbf{\Phi}} \in \mathbb{C}^{N\times N}$ is the quadratic phase modulated on an image. Multiplying it with $\boldsymbol{\rho}$ corresponds to a convolution that generically spreads the spectrum of [3]. The modulation intensity is defined as $h = (\Delta O \cdot T_{\text{enco}})/N$ since the phase modulation can be easily controlled by $\Delta O \cdot T_{\text{enco}}$ (Fig. 2). For the randomly undersampled data, the compressed sensing MRI [1] is adopted to reconstruct the image

$$\hat{\boldsymbol{\rho}} = \arg\min_{\boldsymbol{\rho}} \left\{ \frac{\lambda}{2} \left\| \mathbf{s} - \mathbf{UF}\tilde{\mathbf{\Phi}}\boldsymbol{\rho} \right\|_2^2 + \left\| \mathbf{\Psi}^H \boldsymbol{\rho} \right\|_1 \right\} \qquad (3)$$

where $\lambda$ trades data consistency with sparsity using transform $\mathbf{\Psi}^H$. Numerical algorithm is in [6]. The shift-invariant discrete wavelet is adopted as the sparsifying transform [7].

**Results:** In experiments, the full phase decoding number $N$=256 and the excitation duration $T_{\text{enco}}$=4ms. The field-of-view along Y-axis is $L_Y$=80mm. The repetition time is 1s and the echo time is 20ms. The acquisition rate is 50kHz. The acquired k-space data are shown in Figs. 3(a) and (b), which are undersampled in simulation according to the sampling pattern in Fig. 3(c). Less image features are lost in reconstruction (Fig. 3(f)) using the RF-based spread spectrum than those of using conventional imaging method [1] excluding spread spectrum (Fig. 3(i)).

**Conclusions:** A spread spectrum using chirp RF pulses is proposed and applied in compressed sensing MRI. The intensity of spectrum can be easily controlled by setting bandwidth. Simulation on the sampled data implies that the reconstruction error will be reduced when a proper bandwidth is provided. Further improvement is expected to combing the spread spectrum with adaptive sparse representation [7].

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**References**

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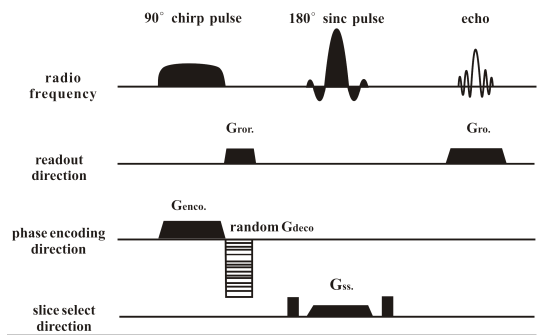


**Fig. 1** Pulse sequence for chirp RF pulses-based spread spectrum MRI

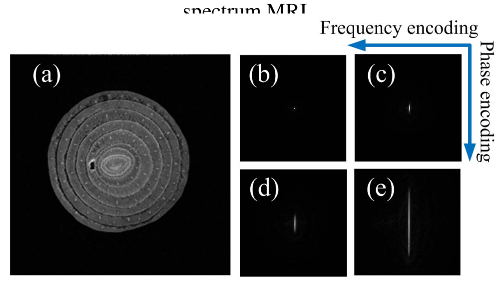


**Fig. 2.** Intensity of k-space for onion data at different chirp pulse bandwidth $\Delta O$. (a) magnitude image, (b)-(e) are the intensity of k-space when $\Delta O$ are 0kHz ($h$=0), 32kHz ($h$=0.125), 64kHz ($h$=0.25) and 256kHz ($h$=1), respectively. Note: $N$=256 and $T_{\text{enco}}$=4ms.

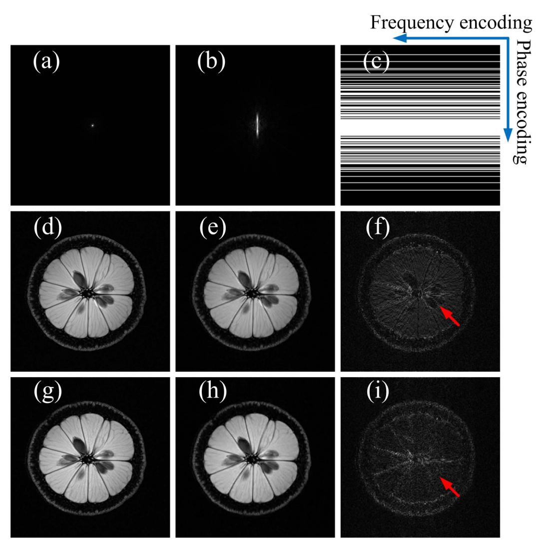


**Fig. 3.** Reconstructed images for lemon. (a) and (b) are fully sampled k-space when $\Delta O$ is 0kHz ($h$=0) and 64kHz ($h$=0.25); (c) is the sampling pattern; (d) and (e) are reconstructed image of fully and 40% sampled data when $\Delta O$= 0kHz; (g) and (h) are reconstructed image of fully and 40% sampled data when $\Delta O$= 64kHz; (f) and (i) are the reconstruction error using undersampled data when $\Delta O$ is 0kHz and 64kHz.