

Field Assessment for Matrix Gradient coils using SVD

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INTRODUCTION: Novel concepts for shimming and encoding using a high number of individual coil elements have been introduced recently (1). It has been shown that using a system with many independent coil elements, the resulting field compositions are not bound to limits of fields generated by a composition of spherical volume harmonics which are generally used for shimming (2). Combining the possibilities of such matrix (gradient) coil systems with new concepts of nonlinear encoding brings up the question of assessing the efficiency of such generalized spatial encoding magnetic fields (SEMs). In this abstract we demonstrate the advantages of the Singular Value Decomposition (SVD) to find efficient SEMs in arbitrary sub volumes for encoding in MRI. This abstract aims for MR scientists working in the field of novel encoding concepts.

METHODS: An arrangement of many individual current carrying wire segments or coil elements can be used to generate a wide variety of magnetic fields. An example of possible combinations of current carrying elements on a cylindrical coil arrangement is shown in Figure 1. Simulations were performed for a cylindrical coil with an inner diameter 0.34m, a length of 0.5m and 8 angular by 11 wire segments along z. Comparisons were made to a monopolar arrangement with a size of 0.42m by 0.5m along and z, respectively. Each current carrying element contributes to the resulting field in each voxel in the volume of interest (VOI). This dependence can be written as a real valued matrix M with the size of the number of current carrying elements m by the number of voxels n . Field compositions along with their significance are calculated using a Singular Value Decomposition (SVD) of the matrix M :

$$M = U\Sigma V^*$$

In this factorization U (with the size $m \times m$) is a unitary matrix, Σ is a diagonal matrix with nonnegative real numbers on the main axis which are the square roots of the eigenvalues and determine therefore directly the significance of the corresponding field component. V^* gives the current distribution of all significant SEMs generated by the SVD. The factorization using the SVD was performed for the following shapes of the target volumes: Sphere, cube, disc and plane. Each target volume was scaled to a large and small target volume with diameters, respectively length of 0.3m and 0.1m. The small volumes were shifted by 0.15m within the physical boundaries of the simulated matrix gradient coil system. Local orthogonality of manually selected slice groups of three encoding fields was calculated using $((|\vec{B}_{z,1}|) \times (|\vec{B}_{z,2}|) \cdot (|\vec{B}_{z,3}|))$. The resulting value depends on the volume spanned by the normalized vectors B_z . All simulations were performed in Matlab (Mathworks, Natick, USA).

RESULTS: Analyzing the shape of the most significant SEMs, a field very similar to Z^2 expressed in spatial volume harmonics has always the highest eigenvalue and therefore the highest significance. For a cylindrical setup the following 3 SEMs after the Z^2 -field have lower eigenvalues by roughly a factor of 3 and are extremely similar to linear SEMs with slight variations along the z-axis. Reducing the FOV to a smaller spherical sub volume does not change the field shape significantly. The fields resulting from the SVD of a centered sphere have close similarity to spherical volume harmonics. By shifting this small spherical VOI off-center the symmetry property is broken and SEMs closer to the coil surface show higher magnitudes which leads to higher gradients in those areas. The result SEMs are no longer easily comparable to spherical volume harmonics. By expanding the spherical VOI to a cubic volume a gradient in the corners is enforced and results in sheering of the fields. For shifting such a cubic volume off-center the same behavior as with the spherical volume can be observed. Reducing a sphere to a disk shaped slice or a cube to one slice results in very similar field shapes to the not reduced ones. A set of three fields can be manually picked with very good orthogonal behavior with one exception: The first and strongest Z^2 SEM has no corresponding orthogonal sub set. A subset of one slice is exemplary shown in Figure 3.

DISCUSSION:

The SVD is very powerful tool to generate meaningful current distributions and therefore encoding fields. The ordering for eigenvalues already gives a ranking for the encoding capability. This method is not restricted to the field shapes used here. Assessing fields for off centered VOI or field components for curved slices could be an application. Engineering problems of matrix gradient coils are solvable to open up new perspectives for novel encoding strategies.

REFERENCES: (1) Juchem et al.; JMR 204, 2010, (2) Juchem et al.; Magn Reson Med 66, 2011

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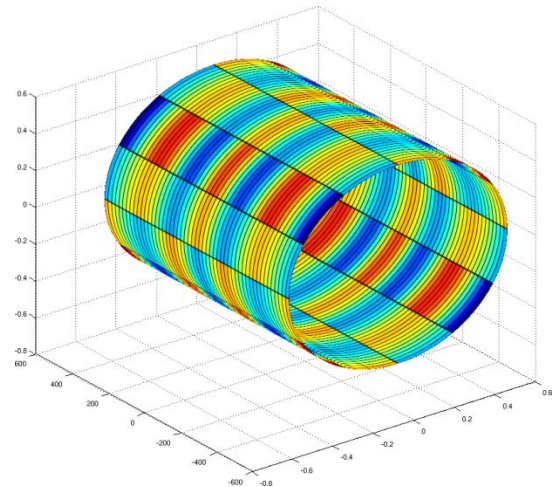


Figure 1: Exemplary representation of a matrix gradient coil

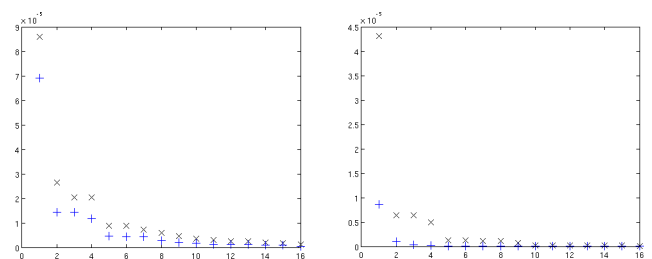


Figure 2: Singular values of Σ which correspond to the eigenvalue and determine the significance of the corresponding field. a) (left) small cubic volume and b) (right) small spherical VOI. '+' corresponds to VOI in the isocenter and 'x' to a VOI shifted off-center

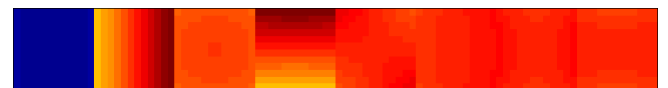


Figure 3: Exemplary field shape of a central cut through the first 8 significant fields generated by SVD