Characterizing the Inherent and Noise-Induced Errors in Actual Flip Angle Imaging

M Louis Lauzon[1,2] and Richard Frayne[1,2]
[1]Radiology, Hotchkiss Brain Institute, University of Calgary, Calgary, AB, Canada, [2]Seaman Family MR Research Centre, Calgary, AB, Canada

Target Audience: Researchers interested in measuring and characterizing the flip angle term in an image.

Purpose: T_1 mapping requires knowing the flip angle, θ, or equivalently $\cos\theta$; it can be determined using the TR-interleaved spoiled gradient-recalled echo AFI (actual flip angle imaging) sequence.[1] AFI makes an approximation and uses non-linear processing, two issues that can affect both the accuracy and precision of the calculation. Our objective is to theoretically determine these errors for $\widehat{\cos\theta}$ (*i.e.*, the estimate of $\cos\theta$) as a function of T_1 and the various AFI acquisition parameters in order to provide an overall error.

Methods: AFI signals S_1 and S_2 are processed to give $\widehat{\cos\theta} = (Rn-1)/(n-R)$, where $R = S_2/S_1$ and $n = \mathrm{TR}_2/\mathrm{TR}_1$. The aforementioned AFI approximation leads to an *inherent* bias, whereas noise produces an *uncertainty* bias; the *net* bias is the sum. Noise also leads to an increase in the variance (σ^2); the relative bias (a measure of accuracy) of random variable z is $\Delta z/z$, and its relative standard deviation (a measure of precision) is σ_z/z. These can be determined analytically using uncertainty analysis.[2,3] All work was done using Matlab (8.1 R2013a; MathWorks, Natick, MA). The theoretical results were verified numerically (*i.e.*, via Monte Carlo simulation) by generating 10^5 instances at each T_1 value, adding normally distributed noise, then calculating the mean and standard deviation (SD) accordingly. The maximum theoretical SNR ($= M_0/\sigma_S$) was set to 1000. The overall error (to roughly 95% confidence) is approximated by the absolute value of the net bias plus two standard deviations.

Results: The relative inherent bias of $\widehat{\cos\theta}$ is $(Rn-1)/[\cos\theta\,(n-R)] - 1$; this error decreases as T_1 increases (Figure A). The relative uncertainty bias is given by $(n^2-1)(1+Rn)/\left[(n-R)^2(Rn-1)\mathrm{SNR}_{S_1}^2\right]$, which increases with T_1 (Figure B), in contra-distinction to the inherent bias. The relative SD of $\widehat{\cos\theta}$ is given by: $(n-R)(Rn-1)/\left[\,\mathrm{SNR}_{S_1}(n^2-1)(1+R^2)^{1/2}\right]$ (Figure C). Relative overall error estimates are provided in Figure D.

Discussion: The inherent bias of $\widehat{\cos\theta}$ decreases as θ or TR_1 decrease, or as T_1 increases since the AFI approximation is $\mathrm{TR}_{1,2} \ll T_1$. The net bias is the superposition of the inherent and uncertainty biases, so different $(n, \mathrm{TR}_1, \theta)$ yield varying proportions. The SD of $\widehat{\cos\theta}$ is essentially the SD of S_1 since n and R are constant with respect to T_1 for a given $(\mathrm{TR}_1, \mathrm{TR}_2, \theta)$. The overall error of $\widehat{\cos\theta}$ shows a significant dependence on (TR_1, θ), and generally increases with T_1 (contrary to the inherent bias).

Conclusion: The analysis herein allows one to tailor the AFI parameters to characterize inherent *vs*. noise-induced errors and minimize overall error. It provides a theoretical and best-case scenario of the error associated with the estimated $\cos\theta$ maps.

References: [1]Yarnykh, Magn Reson Med 2007;57:192-200. [2]Ku, J Res Nat Bur Stand (Eng & Instr) 1966;70C:263-273. [3]Meyer, "Data Analysis for Scientists and Engineers", Wiley Series, New York: John Wiley & Sons, Inc., 1975.

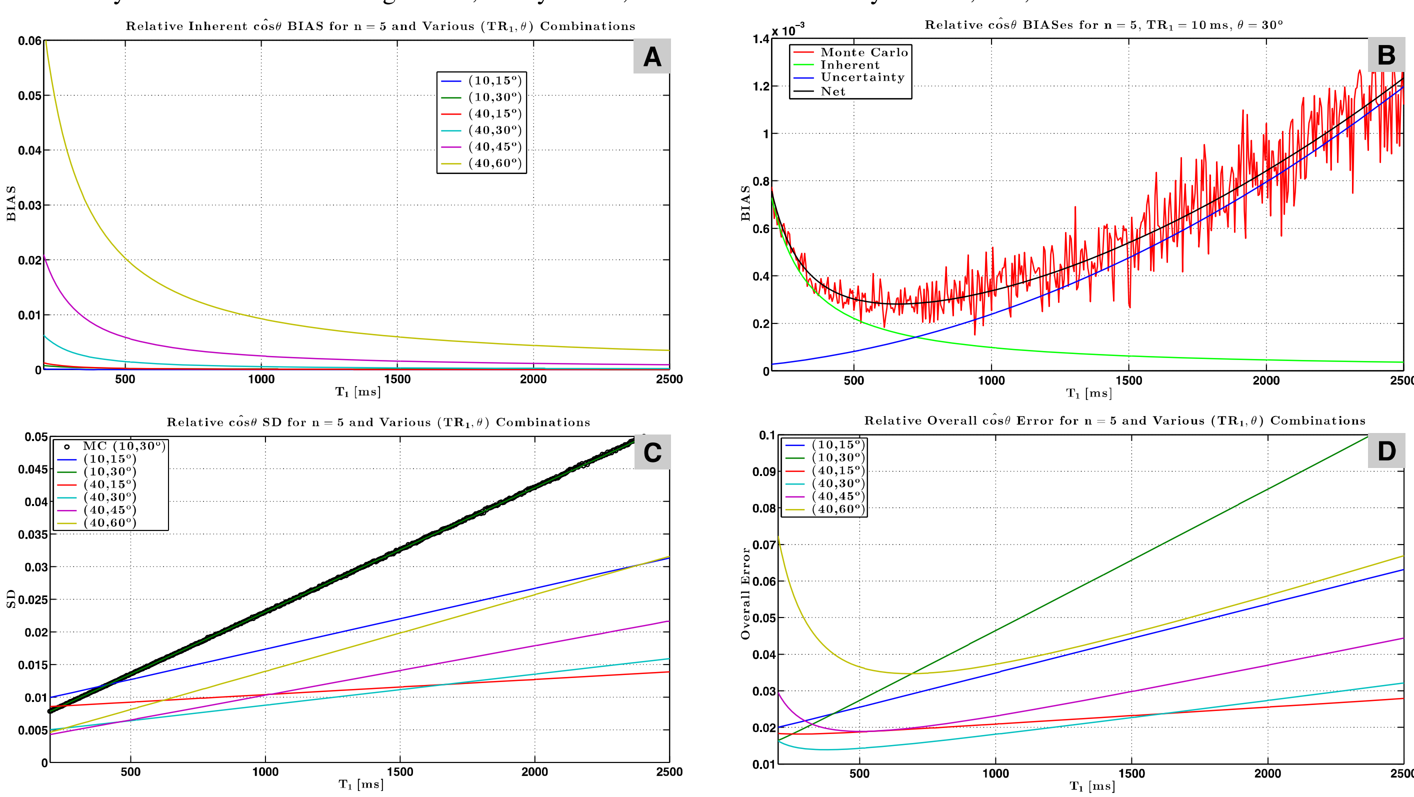

Figure: Analytical relative $\widehat{\cos\theta}$ inherent bias (A), Monte Carlo/inherent/uncertainty/net biases (B), standard deviation (C), and overall error (D) versus T_1 for $n = 5$ and various combinations of (TR_1, θ). The y-axis has been capped to 5% in (C) and 10% in (D).