On variant strategies to solve the Magnitude Least Squares optimization problem in parallel transmission RF pulse design and under strict SAR and power constraints

Nicolas Boulant[1], Andres Hoyos-Idrobo[1], Pierre Weiss[2], Aurelien Massire[1], and Alexis Amadon[1]
[1]*Neurospin, CEA, Saclay, Ile de France, France,* [2]*ITAV, CNRS, Toulouse, Midi-Pyrénées, France*

Abstract: Parallel transmission (pTX) has been a very promising candidate technology to mitigate the radio-frequency field inhomogeneity in magnetic resonance imaging at ultra-high field. For the first few years, pulse design utilizing this technique was expressed as a least squares problem with crude power regularizations aimed at controlling the specific absorption rate (SAR). This approach being suboptimal for many applications sensitive mostly to the magnitude of the spin excitation, and not its phase, the magnitude least squares (MLS) problem then was first formulated in 2007. Despite its importance and the availability of other powerful numerical optimization methods, this problem yet has been faced exclusively by the pulse designer with the so-called variable exchange (V-E) method[1]. Here, we investigate other strategies and incorporate directly the SAR and power constraints. Different schemes such as sequential quadratic programming (SQP), interior point (I-P) methods, semi-definite relaxation (SDR) and magnitude squared least squares (MSLS) relaxations are studied in the small and large flip angle (FA) regimes with B_1 and ΔB_0 maps obtained in-vivo on a human brain at 7 Tesla.

Theory: The examples studied here are 3D excitations using the k_T-points method[2] and targeting 30° and 180° flip angles in the human brain at 7 Tesla with 5 and 7 k_T-points respectively. SAR constraints are enforced by using virtual observation points[3] (VOPs) and the generic head model described in Massire et al[4]. For N_c (= 8) channels, N_{k_T} points (= 5 or 7) and N_{VOPs} (= 490) VOPs, the FA MLS homogenization problem under SAR and power constraints is:

$$\min_x f(\boldsymbol{x}) = \| |g(\boldsymbol{x})| - \theta \|_2^2, \qquad \text{s.t.} \begin{array}{l} c_i(\boldsymbol{x}) \le 10\ \text{W/kg},\ i = 1, \dots, N_{VOPs}, \\ c_G(\boldsymbol{x}) \le 3.2\ \text{W/kg} \\ c_{pw,k}(\boldsymbol{x}) \le 10\ \text{W},\ k = 1, \dots, N_c \\ c_{A,j}(\boldsymbol{x}) = |x_j|^2 \le 1,\ j = 1, \dots, N_c N_{k_T}. \end{array} \tag{1}$$

c_i, c_G, $c_{A,j}$ and $c_{pw,k}$ denote the 10-g SAR constraints over the VOPs, the global SAR constraint, the amplitude constraints and the average power for the k^{th} channel (here taken as 10 W) respectively. For the inversion pulse however the limits were arbitrarily taken as 3 W/kg, 1 W/kg, 2 W and 1 because such pulses are often combined with a long train of small FA pulses. Above θ denotes the target FA (in rads), the function g is either a linear mapping for the small FA problem or a non-linear one computed by integration of the Bloch equation for the large FA problem. The vector $\boldsymbol{x}$ is the concatenated RF waveforms of the N_c channels.

Methods: Some of the most successful large scale algorithms for generally constrained non-linear optimization fall into one of two categories[5]: active-set (A-S) SQP methods and I-P methods. Their implementation was conducted using the optimization toolbox of Matlab (The Mathworks, Natick, MA, USA) along with a BFGS[5] update of the Hessian of the Lagrangian. However, due to the fact that the objective function in (1) is not everywhere differentiable, the variant but closely related MSLS problem $\min_x f(\boldsymbol{x}) = \| |g(\boldsymbol{x})|^2 - \theta^2 \|_2^2$, still under constraints, was also investigated. This time the Hessian of the Lagrangian could be user-supplied by using an analytical formula in the small FA regime or via finite-differences in the large FA case. The Knitro solver (Ziena optimization LLC, Evanston, IL, USA) was used in this particular case. For the design of the inversion pulse, $g(\boldsymbol{x})$ was computed using CUDA and an NVIDIA GPU Tesla card. Initialization of these algorithms was performed using 1) random guesses, 2) the V-E method (varying the power regularization) and 3) SDR.

Results: Simulated FA distributions along with the best normalized root mean square errors (NRMSE) obtained among all initializations/algorithms are illustrated in Fig. 1. Despite the theoretical guarantees when there are no constraints[6], SDR brought little advantage and took several hours of execution. The A-S appeared the fastest, most efficient and most robust (with respect to initialization) method for the small FA problem. Whereas initialization with the V-E method generated cross-correlations between the different input vectors between 0.65 and 1, the correlations obtained for the output vectors varied between 0.98 and 1, indicating convergence towards the same result. Sensitivity with respect to initialization was on the other hand larger for the large FA case. Best NRMSEs were 5.65 % and 8.72 % in the small FA and large FA regimes, by using the MLS and MSLS formulations respectively and the A-S algorithm. Execution time was 6 seconds for the former and 20 min for the latter. Exact knowledge of the Hessian of the Lagrangian however seemed not to be required so that an almost equivalent result of 8.90 % for the inversion pulse could be obtained in 47 s using the approximate but much faster BFGS update method. This NRMSE is larger than our 6 % result obtained using an optimal control approach[7] but is quite remarkable considering the substantially lower number of degrees of freedom optimized (112 versus 96000), a shorter pulse (3.5 ms versus 5.9 ms) and the fact that the constraints this time were directly enforced.

Conclusion: The results imply that the MLS pulse design in pTX under strict constraints of the inversion and small FA pulses needed for an MP-RAGE can be done reliably in under a minute by using the A-S algorithm combined with a V-E initialization (with large power regularization) in both cases. Results also indicate that tackling this particular problem using this approach is more efficient than by looping over least-squares problems as is done with the V-E method[8].

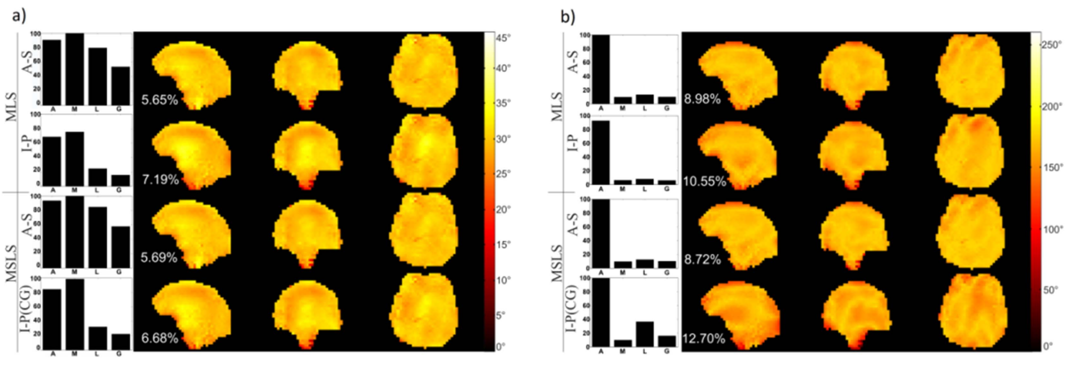

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Fig. 1: FA distributions obtained for the different algorithms in the a) small (30°) and b) large (180°) FA regimes, and with bar-graphs indicating the relative saturations of the different constraints ("A" for amplitude, "M" for maximum average power among the channels, "L" for 10-g SAR and "G" for global SAR).

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