RF Pulse Design using Linear and Nonlinear Gradient Fields: A Multi-Dimensional k-Space Approach

Emre Kopanoglu[1], Leo K. Tam[1], and Robert Todd Constable[1]

[1]*Dept. Diagnostic Radiology, Yale University School of Medicine, New Haven, Connecticut, United States*

Audience: Clinicians/Scientists interested in localized field-of-view (FOV) imaging.

Purpose: This study investigates the excitation fidelity when the number of excitation k-space dimensions is higher than the number of spatial dimensions.

Background: Nonlinear gradient fields (NLGFs) have attracted increased attention in the recent years due to the advantages such fields offer including; spatially varying resolution[1], faster data acquisition[2], reduced radiofrequency (RF) power[3], reduced field-of-view imaging[4] and curved slice imaging[5]. The spatial encoding functions (SEFs) generated by NLGFs have spatially non-uniform variations. In this case, each SEF corresponds to a distribution in conventional k_x-k_y-k_z-space. To describe such a system scientists have used either local k-space representations[6], or nonlinear coordinate systems[3]. Recently, it was shown that when the number of gradient fields exceeds the number of spatial coordinates, either a predefined k-space trajectory can be used[6], or a multi-dimensional k-space can be utilized for trajectory design[7], with both approaches defined for encoding purposes rather than excitation.

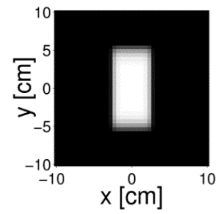

Figure 1: Target excitation profile.

In this study, the effect of increasing the number of gradient fields beyond the number of spatial coordinates on excitation, specifically on a multi-dimensionally selective RF pulse[8], is investigated. As the number of fields to choose from increases, it becomes less trivial to predefine a trajectory. Therefore, a similar approach to Ref. 7 is used, rather than Ref. 6. Although the method suggested in Ref. 7 is suboptimal for encoding purposes, since it requires *a priori* information of the image to be obtained, it is applicable to excitation pulse design since the required information for excitation is the target profile.

Methods: Simulations are performed using Matlab (Mathworks, Natick, MA, USA), on a 40x40 grid on the xy-plane with FOV_x=FOV_y=20cm. The sampling distance in the excitation k-space is set to $1/FOV_x$ for the LGFs and $1/FOV_x^2$ for the NLGFs, so that maximum $\partial B/\partial t$ is the same inside the FOV, and the number of SEFs generated by each field is set to 40. The target profile is shown in Figure 1.

RF pulse design is performed iteratively. In each iteration, the SEF that contributes the most to the excitation profile is selected using the Matching-Pursuit (MP) algorithm[9], added to the set of selected SEFs, and removed from the pool of SEFs available for the next iteration. Then, using the set of selected SEFs, the RF pulse is optimized using the Conjugate-Gradient (CG) algorithm[10]. Next iteration starts after the residual profile, which is obtained by subtracting the profile generated by the RF pulse from the target profile, is set as the target of MP. The number of CG iterations is set to the number of selected SEFs. As a measure of excitation fidelity, root-mean-squared-error (RMSE) is used[3]. The maximum number of selected SEFs is 100.

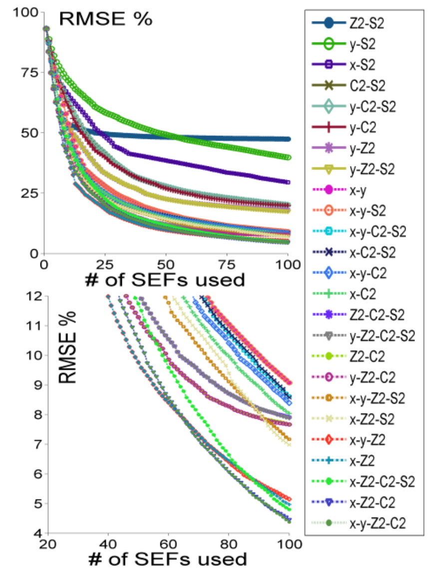

Figure 2: RMS-error versus the number of SEFs used in RF pulse design. (**Top**) All gradient sets. (**Bottom**) Gradient sets that yield lower RMSE than the x-y gradients. The sets are sorted with respect to the lowest RMSE they yield.

Results: Fig. 2 shows how RMSE changes for the simulated cases. Of the 16 cases, which yield a lower RMSE than the LGFs, 8 use a 3D k-space and 5 use a 4D k-space, whereas only 3 cases used a 2D k-space. Furthermore, the lowest RMSE is obtained using a 4D k-space, demonstrating the benefit of increasing the number of k-space dimensions beyond the number of spatial coordinates.

Even though the profile is rectangular, which can most easily be defined as a function of x and y, rather than the Z2, C2 and S2 harmonics; NLGFs are highly efficient in encoding the target profile: of the 10 cases with the lowest RMSE, all utilize the Z2, whereas x, C2, y and S2 are utilized 7, 6, 4 and 2 times, respectively. Furthermore, Figure 3 shows that for the lowest error cases, SEFs generated using the Z2-field are utilized while transmitting most of the RF power, whereas the y gradient is nearly never used. The power levels suggest x, Z2 and C2 are the most useful fields for this FOV definition and a 3D k-space coverage is sufficient for excitation of the target profile. As opposed to the 9.1% RMSE obtained with the LGFs, the lowest RMSE is 4.4%, obtained using x-y-Z2-C2 fields (Fig. 4) whereas the x-Z2-C2 case yields 4.5% RMSE. Furthermore, the lowest RMSE that was obtained using the LGFs, which required 100 SEFs, can be obtained using the x-y-Z2-C2 set with only 57 SEFs. In this case, the NLGF set requires 16.3% higher RF power, although the lower number of SEFs suggests a 33.7% lower normalized-SAR[3] (neglecting slew-rate and gradient amplitude limits).

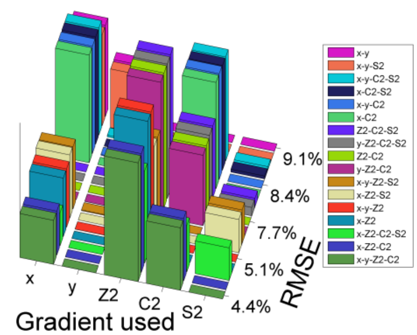

Figure 3: RF power deposited in excitation k-space for the gradient sets given in Figure 3, using 100 SEFs. The height of the bars along "Z2" represents how much power is deposited in regions with $k_{z2} \neq 0$ (similarly for other fields).

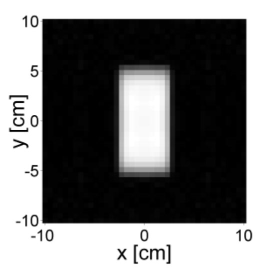

Figure 4: Profile obtained using x-y-Z2-C2 fields and 100 SEFs.

Although adding a field to the set of available fields should not increase the RMSE, some cases indicate otherwise: x-Z2 yields a lower error than x-y-Z2 and x-Z2-S2. This is because MP is a greedy algorithm, and it may not always converge to the global minimum of the optimization problem. Although a brute-force search algorithm can be used instead, computational cost would increase significantly, since there are $\binom{2{,}560{,}000}{N}$ possible SEF combinations in the 4D k-space when selecting N SEFs. Figure 5 demonstrates that when NLGFs are used, predefining a k-space trajectory is not as straightforward as when only LGFs are used.

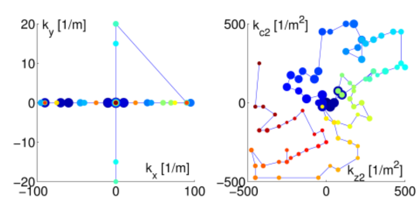

Figure 5: k-Space locations of the selected SEFs. The SEFs are ordered using a travelling salesman algorithm[11], and then color coded. Larger markers indicate higher RF power deposition in the corresponding k-space location.

Discussion and Conclusion: In this study, the effect of NLGFs on excitation fidelity is investigated using two linear and three nonlinear gradient fields. By treating the spatial distributions generated using NLGFs as lying on distinct spatial frequency coordinates, the number of k-space dimensions is increased beyond the number of spatial coordinates. It is shown on a 2D selective pulse design example that such an approach may yield RF pulses that increase excitation fidelity.

The target profile is selected to be rectangular, which arguably is one of the most important 2D selective profiles, when LGFs are used for encoding. Even though the profile shape is more compatible with the LGFs, it is shown that better excitation profiles can be obtained using NLGFs. This is because NLGFs offer spatially non-uniform encoding capabilities: for example, the Z2 field varies slowly inside the excitation region where the profile is flat, and rapidly around the transition regions of the profile, which makes it more suitable to encoding both regions simultaneously.

Acknowledgments: NIH BRP R01 EB012289-01. **References:** (1) Hennig J, MAGMA 2008;21(1-2):5-14. (2) Stockmann JP, MRM 2010;64(2):447-456. (3) Kopanoglu E, MRM 2013;70(2):537-546. (4) Kopanoglu E, Proc. ISMRM, 2012; p. 3471. (5) Weber H, DOI: 10.1002/mrm.24364. (6) Gallichan D, MRM 2011;65(3):702-714. (7) Lin FH, MRM 2013;70(1):86-96. (8) Bottomley PA, J Appl Phys 1987; 62(10):4284-4290. (9) Mallat and Zhang, IEEE Trans. Signal Process, 41:3397-3415 (1993). (10) Hestenes, et al., J Res Nat Bur Stand, 49:409-436 (1952). (11) Kirk J. Traveling Salesman Problem - Genetic Algorithm. 2009.