Noise Reduction of Impulse Response Function of the Encoding Fields Calculation

Frederik Testud[1], Johanna S. Vannesjö[2], Christoph Barmet[2,3], Klaas P. Pruessmann[2], Jürgen Hennig[1], and Maxim Zaitsev[1]

[1]*Medical Physics, Department of Radiology, University Medical Center, Freiburg, Germany,* [2]*Institute for Biomedical Engineering, University and ETH Zürich, Zürich, Switzerland,* [3]*Skope Magnetic Resonance Technologies, Zürich, Switzerland*

Introduction The **I**mpulse **R**esponse **F**unction of **E**ncoding **F**ields (**IREF**s) $\boldsymbol{h}(t)$ characterizes the complete encoding hardware chain, starting with the console, where the **S**patial **E**ncoding **M**agnetic fields (**SEM**) waveform $g(t)$ is programmed, all the way to the produced SEMs, assuming linear behavior of all components[1,2]. $\boldsymbol{h}(t)$ or its **F**ourier **T**ransform (**FT**) $\widetilde{\boldsymbol{H}}(\omega)$ can be used for improved preemphasis[1,2] or image reconstruction[3]. $\widetilde{\boldsymbol{H}}(\omega)$ can be estimated using **M**agnetic **F**ield **M**onitoring (**MFM**)[2,4] which describes the image encoding hardware in terms of S real-valued spherical harmonics, so self- and cross-terms are assessed. The proposed methods[1,2] for estimating $\widetilde{\boldsymbol{H}}(\omega)$ suffer from increased noise at high frequency and need to be low-pass filtered which might affect the accuracy of IREF determination. In this work, we propose an improved IREFs calculation by taking into account the digitization of the input waveform and improving the differentiation of the measured trajectory $\vec{k}(t)$.

Theory The output $G(t)$ of a linear system can be modeled as a convolution of the input $g(t)$ with the system's impulse response function $h(t)$, Eq. [1]. Here, the system consisted of C SEM coils and N triangular shaped waveforms ('blips') were used for $g(t)$ [2]. The time integral of the output $G_{c,s,n}(t)$ ($c = 1 \ldots C$, $s = 1 \ldots S$, $n = 1 \ldots N$) $k_{c,s,n}(t)$ is calculated in the **L**east **SQ**ua**R**e sense (**LSQR**) from the measured probes data of a dynamic field camera[2,5]. The elements $\widetilde{H}_{c,s}(\omega)$ of $\widetilde{\boldsymbol{H}}(\omega)$ (size $C \times S$) are calculated from the FT inputs $\tilde{g}_{c,n}(\omega)$ ($\bar{\tilde{g}}$ being the complex conjugate of $\tilde{g}$) and outputs $\widetilde{G}_{c,s,n}(\omega)$, Eq. [2]. In the previously published IREF estimations[1,2], $g_{c,n}(t)$ is linearly interpolated to the acquisition dwell time and $G_{c,s,n}(t)$ is obtained via point-by-point differentiation of $k_{c,s,n}(t)$. Alternatively, $G_{c,s,n}(t)$ can be calculated in the LSQR sense by obtaining the 1st derivative of the field probes' phase $\vec{\Phi}(t)$ using the adaptive **S**avitzky-**G**olay (**SG**) filter[7]. SG approximates M points of $\vec{\Phi}(t)$ with a polynomial of maximal order K.

$$G(t) = \int_0^t h(t-\tau) g(t) d\tau \qquad [1a]$$

$$\Updownarrow$$

$$\tilde{G}(\omega) = \tilde{H}(\omega)\tilde{g}(\omega) \qquad [1b]$$

$$\tilde{H}_{c,s}(\omega) = \frac{\sum_{n=1}^{N} \tilde{G}_{c,s,n}(\omega)\bar{\tilde{g}}_{c,n}(\omega)}{\sum_{n=1}^{N} \tilde{g}_{c,n}(\omega)\bar{\tilde{g}}_{c,n}(\omega)} \qquad [2]$$

Methods The IREFs of C=3 linear SEMs from a 3 T scanner (TRIO a Tim System, Siemens AG, Germany), with activated eddy current and delay correction, were estimated with a dynamic field camera operated in transmit/receive mode[5,6]. 16 proton-based field probes were approximately distributed on an 18 cm sphere strapped to the patient table. 49 repetitions of N = 16 blips (slew rate = 153 T/m/s, amplitudes varying from 7.65 to 30.6 mT/m in steps of 1.53 mT/m, SEM raster time Δ_T = 10 µs) were acquired with a repetition time of TR = 2 s for averaging. The field evolution was acquired for 50 ms. Data reconstruction and analysis were performed offline in MATLAB (The Mathworks, Natick, AM, USA). The parameters of the SG-filter were M = 100 and K = 20. The IREFs of the linear SEM$_c$ ($c = x, y, z$) coils were estimated in three ways: $\widetilde{H}_{c,s}^{(a)}(\omega)$ using point-by-point differentiation and linearly interpolated input $g_{c,n}(t)$; $\widetilde{H}_{c,s}^{(b)}(\omega)$ using the probes' phase derivative obtained by SG filtering and linearly interpolated input $g_{c,n}(t)$; $\widetilde{H}_{c,s}^{(c)}(\omega)$ using the probes' phase derivative obtained by SG filtering and the quantized input SEM waveform $g_{c,n}(t)$.

Results In Fig. 1 $\left|\widetilde{H}_{x,x}^{(a)}(\omega)\right|$, $\left|\widetilde{H}_{x,x}^{(b)}(\omega)\right|$ and $\left|\widetilde{H}_{x,x}^{(c)}(\omega)\right|$ are shown exemplarily for the SEM$_x$ coil. Most of the high frequency noise is removed in $\left|\widetilde{H}_{x,x}^{(b)}(\omega)\right|$ compared to $\left|\widetilde{H}_{x,x}^{(a)}(\omega)\right|$. The noise level and the 1st and 2nd harmonic amplitudes of Δ_T (100 kHz and 200 kHz) are reduced in $\left|\widetilde{H}_{x,x}^{(c)}(\omega)\right|$. The self-terms' magnitude and phase of the SEM$_x$, SEM$_y$ and SEM$_z$ coils are shown in Fig. 2. The -3 dB bandwidth of the IREFs can be easily estimated from $\left|\widetilde{H}_{c,c}^{(c)}(\omega)\right|$. It is 21.4 kHz, 20.1 kHz and 22.5 kHz for the SEM$_x$, SEM$_y$ and SEM$_z$ coils, respectively.

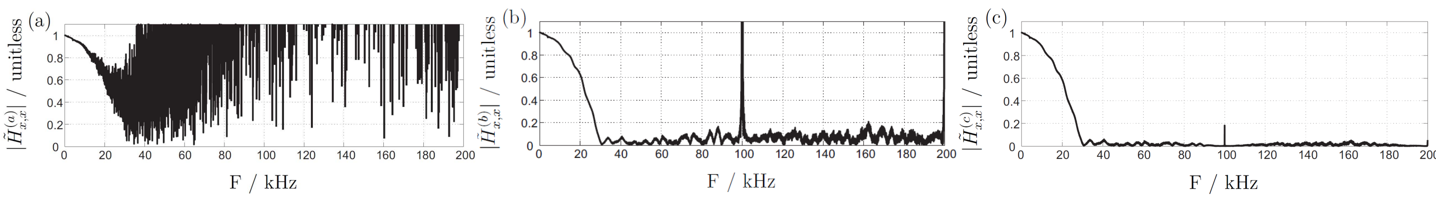

Fig. 1 (a) $\left|\widetilde{H}_{x,x}^{(a)}(\omega)\right|$*; (b)* $\left|\widetilde{H}_{x,x}^{(b)}(\omega)\right|$*; (c)* $\left|\widetilde{H}_{x,x}^{(c)}(\omega)\right|$*. Clear reduction of high frequency noise is visible in* $\left|\widetilde{H}_{x,x}^{(b)}(\omega)\right|$ *and* $\left|\widetilde{H}_{x,x}^{(c)}(\omega)\right|$*.*

Discussion Improved IREFs were obtained using the probes' phase derivative from SG-filtering and the discretized input waveforms (Fig. 1(c)). The method is independent of the input waveform, thus frequency sweeps based IREFs[1,8] will also profit from the proposed estimation. The achieved improvements using discrete input waveforms indicate that the discretized k-space trajectory should be used in image reconstruction. Using the IREFs the waveforms could be predicted without further filtering with the limitation, however, that non-linear effects cannot be taken into account as with concurrent MFM[3]. The proposed calculation steps allow to reduce considerably the noise of the IREFs which potentially can be used to reduce the number of necessary repetitions, in particular if the IREFs of matrix coils with many channels need to be acquried[9].

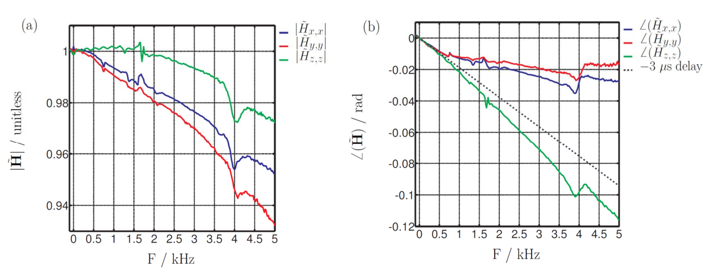

Fig. 2 (a) Magnitude and (b) phase of $\widetilde{H}_{c,c}(\omega)$ ($c = x, y, z$)*. A delay of -3µs is simulated in (b) for comparison.*

References 1. Addy et al., MRM, 68:2012; 2. Vannesjö et al., MRM, 69:2013; 3. Graedel et al., Proc ISMRM, p552, 2013; 4. Wilm B. et al., MRM 65:1690, 2011; 5. Barmet, Proc ISMRM, p781, 2009; 6. Testud et al., Proc ISMRM, p2596, 2012; 7. Barak, Anal Chem, 67:1995; 8. Vannesjö et al., MRM, doi: 10.1002/mrm.24934, 2013; 9. Jia et al. Proc ISMRM, p666, 2013;

Acknowledgments This work was part of the INUMAC project, supported by the German Federal Ministry of Education and Research, grant #01EQ0605 and #13N9208, by the European Research Council Advanced Grant 'OVOC' grant agreement #232908 and by the European Research Council Starting Grant 'RANGEmri' grant #282345.