generalized measure to assess gradient coil performance

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Target audience: Developers of innovative gradient hardware and novel spatial encoding strategies for MRI.

Purpose: To propose a performance measure that is applicable to all types of gradient hardware, including conventional gradient coils and matrix coils.

Background: Over the past 30 years many publications have discussed performance measures for assessing different MRI gradient coils. Most of those performance measures were used to quantify conventional gradient coils that generate linear encoding fields [1-3]. Matrix coils [4,5] can enhance the encoding efficiency, but multiple coil elements and the nonlinear nature of the generated encoding fields require different methods to assess their performance. A novel performance measure was proposed in [5] that measures the local encoding efficiency of a matrix coil. However, the relationship between this type of performance measure and conventional performance parameters was not clear. This work presents this relationship and proposes a general performance measure applicable to all types of gradient hardware. A cylindrical matrix coil is used to illustrate the usefulness of this general measure. The generalized measure is tested with a cylindrical matrix coil revealing novel features of matrix coil designs.

Methods: To propose a general performance assessment, we consider a hypothetical matrix coil [5]. Figure 1 shows a schematic representation of a cylindrical matrix coil with 8x21 elements. Each element consists of a gradient-generating arc segment and two short radial segments for connections. The focus in the design of the coil is to validate the proposed general performance assessment rather than to generate a viable coil construction; therefore, this simplified model is appropriate.

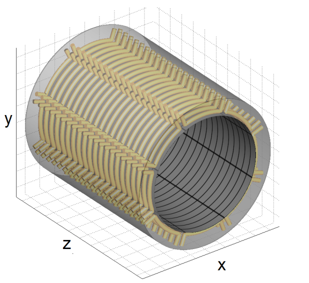

Figure 1: Schematic representation of an 8x21 cylindrical matrix coil.

To propose a generic performance parameter, a figure of merit of conventional linear gradient coils is investigated first. For a linear gradient coil, the resistive figure of merit $\eta/R^{1/2}$ [3] is often used to measure the coil performance, where R is the coil resistance and η denotes the efficiency. η corresponds to the gradient amplitude G at the origin divided by the current I flowing through the coil, i.e. $\eta = G/I$. As described in [3], a larger value of $\eta/R^{1/2}$, i.e. $G/P^{1/2}$ (with the dissipated power of the coil $P = I^2R$), indicates that the coil has a better performance. Consider next a conventional gradient system consisting of x-, y- and z-gradient coils with gradient strength of G_x, G_y and G_z and dissipated power P_x, P_y and P_z. In view of the measure $G/P^{1/2}$, it seems useful to use $G_xG_yG_z/(P_xP_yP_z)^{1/2}$ to assess the performance of the whole gradient system. Without loss of generality, it is assumed that $P_x = C_x P_z$ and $P_y = C_y P_z$ in the design of a linear MRI gradient system where C_x and C_y are constants. Defining the total dissipated power as $P_{total} = P_x + P_y + P_z$, one finds

$$G_xG_yG_z/(P_xP_yP_z)^{1/2} = C^*G_xG_yG_z/P_{total}^{3/2}, \qquad (1)$$

where $C=(1+ C_x + C_y)^{3/2}/(C_xC_y)^{1/2}$. This formula can now be generalized to matrix coils. First, consider that $G_xG_yG_z$ is proportional to the image resolution that may be achieved with a conventional gradient coil. For non-orthogonal gradient directions, this term may also be written as $G_xG_yG_z \approx (\nabla B_{z1}\times\nabla B_{z2})\cdot\nabla B_{z3}$ because the gradient ∇B_{zi} of the magnetic fields B_{zi} is equivalent to G_i, i=x,y,z. In this form, this equation is also applicable to nonlinear encoding fields, where the gradients may vary from one spatial location to the other. With the definition of a multidimensional encoding function $\mathbf{\Psi} = (B_{z1}, B_{z2}, B_{z3})$ [6], the above term can be written more concisely as $|\det(\partial\mathbf{\Psi}/\partial\mathbf{x})| = |(\nabla B_{z1}\times\nabla B_{z2})\cdot\nabla B_{z3}|$. This term describes the local image resolution for 3D imaging and nonlinear encoding fields. Be aware that local image resolution varies with the spatial coordinate as $\mathbf{\Psi}$ is not a fixed quantity. Therefore, it makes sense to sum $|\det(\partial\mathbf{\Psi}/\partial\mathbf{x})|$ at a set of test points within an ROI, i.e. $F = \sum |\det(\partial\mathbf{\Psi}/\partial\mathbf{x})|$. Based on the above investigation, it is useful to define a generalized performance parameter

$$\beta = F/(n\, P_{total}^{3/2}) \qquad (2)$$

to assess a gradient coil for a given P_{total} and n test points in the ROI. In order to find a matrix coil with optimal performance β, an optimization problem is formulated as follows

$$\max_{I_{i,j}} F,\ F = \sum_{k=1}^{n}\left|\det\left(\frac{\partial\mathbf{\Psi}(\mathbf{x}_k, I_{i,j})}{\partial\mathbf{x}}\right)\right|, \text{subject to } \sum_{i,j} l_j I_{i,j}^2 \le P_{total}, \qquad (3)$$

where $\mathbf{x}_k=(x_k, y_k, z_k)$, k=1, …, n, denote the coordinate vectors of n test points in a ROI, $I_{i,j}$ denotes the current flowing through the j-th coil element when generating the magnetic field B_{zi}, i = 1,...,3, j = 1,…,m (i.e. $B_{zi} = \sum_j b_j I_{i,j}$, where b_j is the sensitivity of the j-th coil element calculated using Biot-Savart's law) and m is the total number of coil elements. In the constraint in (3), l_j denotes the length of the j-th coil element, thus this sets a limitation of the dissipated power in the entire coil. Compared with the optimization problem in [5], the problem (1) does not contain upper and lower bound constraints of current since typically such constraints are also not used in the design of conventional linear gradient coils.

Additional constraints may be introduced into the optimization problem to assess the matrix coil from different perspectives. For example, as shown in [5], current constraints can be added to satisfy the technical requirements of available in-house gradient power amplifiers driving the matrix coil. In this work, in order to assess the capability of the matrix coil to generate linear gradient fields, constraints for linear target fields, i.e. $|B_{z1} - \alpha x| \le 0.05\max(\alpha x)$, $|B_{z2} - \alpha y| \le 0.05\max(\alpha y)$ and $|B_{z3} - \alpha z| \le 0.05\max(\alpha z)$, were added into the optimization problem. Here, α is an extra design variable of the optimization problem in order to guarantee $\sum l_j I_{i,j}^2 = P_{total}$ for the optimal currents $I_{i,j}$. In this example, 8x49 and 16x49 matrix coils, referred to as coil I and II respectively, were used. All the numerical examples were solved with the function fmincon from the optimization toolbox of MATLAB (The MathWorks. Natick, USA) to obtain optimal current flows through the matrix coil elements for a spherical ROI of radius 20 cm centered at the origin. The considered matrix coils had dimensions similar as whole-body gradient coils (inner diameter: 68cm, outer diameter: 88cm, length: 120cm). In all calculations, the total power loss was set to $P_{total} = 5.6e5\ A^2m$.

Results and Discussion: Figure 2 plots optimal currents for coils I (Fig. (a-c)) and II (Fig. (d-f)), respectively. As shown in Figure 2 current distributions for coil II are smoother and the currents maximum become smaller than for coil I. However, β = 2.58e-16 for coil II which is by a factor of 1.31 smaller than for coil I. This phenomenon may be caused by gaps (Fig. 1) between two adjacent coil elements. The increasing number of elements in the circumferential direction increases the total gap lengths, thus possibly leading to the reduction of β.

From the linear gradient field point of view, β can also allow us to compare a matrix coil with a conventional gradient coil system. To find the optimal performance β for the conventional coil system, we can generalize the optimization problem (3) by using the stream function [3] for the current as design variables and adding the constraints concerning linear target fields. By solving the generalized problem, we can calculate β for the conventional coil system and perform a performance comparison with the matrix coil, which is ongoing work.

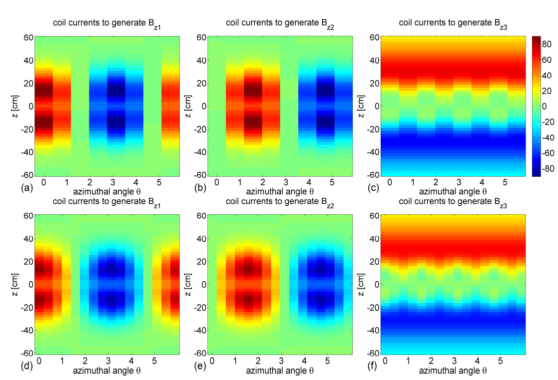

Figure 2: Optimal currents with constraints for linear target fields for the 8x49 (a-c) and 16x49 (d-f) matrix coils.

References: [1] Carlson et al., MRM 1992, 26:191; [2] Turner, J Phys E: Sci Instrum 1988, 21:948; [3] Poole et al., MRM 2012, 68:639; [4] Juchem et al., MRM 2011, 66:893; [5] Jia et al., Proc. ISMRM 2013, #666; [6] Schultz et al., MRM 2010, 64:1390.

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