Eigen matrix approach in coupled-circuit numerical simulation of eddy currents in MRI systems

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Introduction: Eddy currents are inevitable responses of MRI gradient signals. Its characteristic is highly dependent on system structures and signal sequences. To implement efficient compensation techniques we need to know its transient and spatial characteristics on every MRI system. In this work, we have performed coupled circuit simulation [1-3] of eddy currents temporal and spatial responses for both open and closed-bore MRI systems. Eigen matrix techniques of solving differential equations have been implemented in the calculation process to calculate for large number of domains. FID measurement of eddy fields has been conducted by an NMR probe to verify our results. We have found good agreements between experiment and simulation.

Methods and materials: In coupled-circuit method, eddy current conducting structures are divided into multiple layers along thickness. To consider eddy current distribution along length (and/or width) it is needed to divide each layer into slices. Though ideally layer-thickness should be infinitely thin, each layer is taken enough thin to consider constant eddy current [3]. Traditionally thickness is taken around 1/5th of the corresponding skin depth (δ) [1, 3]. Since decreasing the thickness increases efficiency and accuracy of characterizing eddy currents, we have taken thickness to be less than 1/5th of δ. But decreasing the layer thickness increases total number of slices or domains. System of differential equations becomes larger and solving this large number of differential equations takes much longer time, reduces efficiency and provides singularity problems. Coupled differential equation, $\mathbf{M}_{ii}\frac{d\mathbf{I}(t)}{dt}+\mathbf{R}_i\mathbf{I}(t)=-\mathbf{M}_{is}\frac{di_s(t)}{dt}$, where $\mathbf{M}_{ii}$ is matrix of self-inductances L_i and mutual inductances among subdomains, $\mathbf{M}_{is}$ is matrix of inductive couplings between gradient coil and slices, $\mathbf{R}_i$ is resistance matrix of all slices, $i_s(t)$ is gradient coil current and $\mathbf{I}(t)$ is eddy current matrix. To compute inductive couplings between biplanar gradient coil for open MRI system and any other eddy current conducting structure, we have implemented solid-angle form of Ampere's law [4, 5]. For computing all other inductive coupling and self-inductance matrices we followed formulas from Ref [6] and methods of Ref [1]. For a trapezoidal signal, above equation becomes nonhomogeneous for rising- and falling-ramps and homogeneous for constant portion. For these two cases we have implemented Eigen method algorithms of solving system of differential equations:

Case 1: Homogeneous

For initial value homogeneous linear system of $\frac{d\mathbf{I}(t)}{dt}=\mathbf{A}\mathbf{I}(t)$, and $\mathbf{I}(t_0)=\mathbf{I}_0$,

(i) Compute the eigenvalues and eigenvectors of the coefficient matrix $\mathbf{A}=-\mathbf{M}_{ii}{}^{-1}\mathbf{R}_i$

(ii) Use eigenvalues and eigenvectors of $\mathbf{A}$ to, respectively construct diagonal matrix $\mathbf{D}$ and the change of basis matrix $\mathbf{C}$ such that, $\mathbf{I}(t)=\mathbf{Cz}$ &, $\mathbf{D}=\mathbf{C}^{-1}\mathbf{AC} \leftrightarrow \mathbf{A}=\mathbf{CDC}^{-1}$

(iii) Write down general solution of decoupled system $\frac{d\mathbf{z}}{dt}=\mathbf{Dz} \rightarrow \mathbf{z}=\begin{bmatrix}c_1e^{\lambda_1 t}\\ \vdots \\ c_ne^{\lambda_n t}\end{bmatrix}$

(iv) Determine the coefficient matrix $\mathbf{c}=\mathbf{C}^{-1}\begin{bmatrix}I_1(t_0)e^{-\lambda_1 t_0}\\ \vdots \\ I_n(t_0)e^{-\lambda_n t_0}\end{bmatrix}$

(v) The solution of the original (coupled) system will be, $\mathbf{I}(t)=\mathbf{Cz}$

Case 2: Nonhomogeneous

For initial value nonhomogeneous equations, $\frac{d\mathbf{z}}{dt}=\mathbf{Dz}+\mathbf{E}(t)$, $\mathbf{I}(t_0)=\mathbf{I}_0$

where, $\mathbf{E}(t)=\mathbf{C}^{-1}\mathbf{B}\frac{di_s(t)}{dt}$, we implement fundamental matrix method.

(i) Expressed the fundamental matrix as

$$\mathbf{\Phi}(t)=\begin{bmatrix}\boldsymbol{v}_{11}e^{\lambda_1 t} & \boldsymbol{v}_{12}e^{\lambda_2 t} & \ldots.\boldsymbol{v}_{1n}e^{\lambda_n t}\\ \vdots & \vdots & \vdots\\ \boldsymbol{v}_{n1}e^{\lambda_1 t} & \boldsymbol{v}_{n2}e^{\lambda_2 t} & \ldots.\boldsymbol{v}_{nn}e^{\lambda_n t}\end{bmatrix}$$

where, $\boldsymbol{v}_{n1}, \boldsymbol{v}_{n2}, \ldots., \boldsymbol{v}_{nn}$ are the associated eigenvectors, and $\lambda_1, \lambda_2, \ldots., \lambda_n$ are the eigenvalues of the corresponding homogeneous equation.

(ii) The solution of the nonhomogeneous equations can be given by

$$\boldsymbol{I}(t)=\mathbf{\Phi}(t)\mathbf{\Phi}(0)^{-1}\mathbf{I}(t_0)+\int_{t_0}^{t}\mathbf{\Phi}(t)\mathbf{\Phi}(s)^{-1}\mathbf{E}(s)ds$$

To see the effectiveness of this approach, we have done simulation in Mathematica® for both open MRI (0.3 T) and close-bore MRI (9.4 T narrow bore superconducting magnet) systems. Responses of eddy currents in different slices are then implemented in Biot-Savart's law to compute eddy magnetic fields. FID measurements have been conducted using an NMR probe and stepper motor positioning system to verify our simulation results. The experimental setup is shown in Fig. (a) for open MRI system.

Results and discussion: Fig. (b), Fig (c) show spatial fields generated by eddy currents in local shielding box of open MRI system. Gradient signal ramp-up and ramp-down duration was $170\mu s$ and flat-top duration was 1.06ms and corresponding δ was $1.60739mm$. The thickness of brass plates of rectangular box was 0.3 mm. We took two layers along thickness (Z-axis) and 110 slices in each layer both along the length (X-axis) and width (Y-axis). Once we compute all the matrices which take less than 2 minutes, we can use these data for any gradient signals. It took less than one minute to solve overall system of equations. Fig. (d) shows eddy current time constants taken at different points in ROI. We have found a good agreement between simulation and experiments. In case of close-bore MRI system we considered eddy currents due to innermost copper layer. The cylindrical layer is of thickness 2mm with diameter and length 53.84 mm and 978 mm, respectively. The gradient signal was: ramp-up and ramp-down duration $200\ \mu s$ and flat-top duration $600\mu s$, δ was 0.93459 mm. We take 10 layers and 486 slices in each layer. Transient secondary field responses along the Z-axis are illustrated in Fig. (e) and (f), respectively for open and closed-bore MRI systems. We have also calculated cross terms of eddy fields (X-field for Gz coil of open MRI system is shown in Fig. (g)). Our approach is less time-consuming and efficient for simulating eddy currents in MRI systems.

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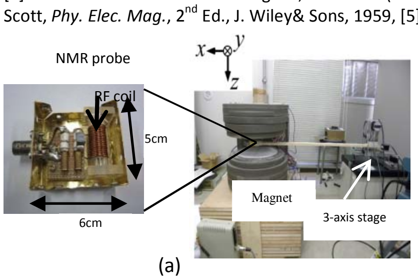
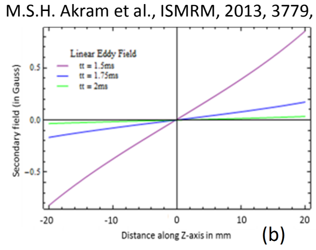
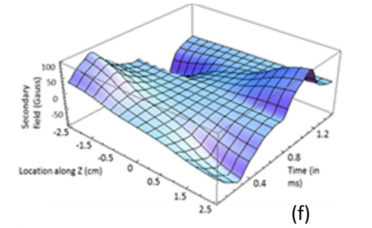


(a)



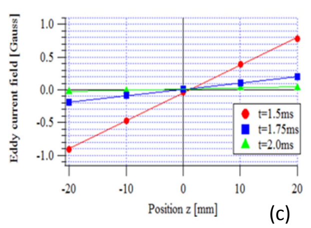
(b)

(d)

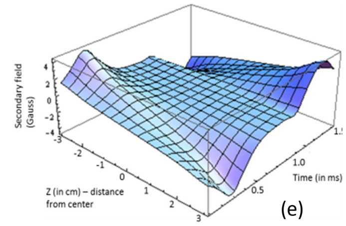


(f)

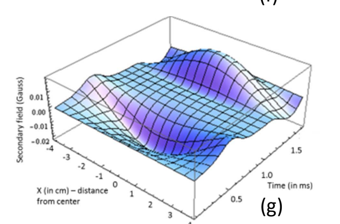
Fig. (a) Experimental setup , (b) Simulated eddy fields along Z-axis, (c) Experimental eddy fields along Z-axis, (d) Eddy field time constants, (e) Transient field for open MRI system along Z-axis, (f) Transient field for closed-bore MRI system along Z-axis , (g) X-fields for Z-gradient coil



(c)



(e)



(g)