Magnetic Force Estimation around MRI Magnets

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Introduction: Magnetic forces around modern MRI scanners with powerful superconducting magnets have long been a major concern for operator and patient safety [1]. These safety concerns range from ferrous material in service tools or medical equipment to implants of patients. It is desired to know the spatial profile of magnetic force distribution which can serve as a guideline for handling ferrous objects around MRI magnets. It is known that the vectorial magnetic forces on ferrous objects depend on both the properties of background magnetic field and the object. The exact calculation of forces can only be achieved on a case by case basis using FEM based method. Therefore the consensus of the industry has been to provide a quantity which is a good estimation and indicator of magnetic forces. This quantity has been included in IEC 60601-2-33[2]. However, the mathematical definition of this quantity has been discussed recently where two suggestions are presented. Namely, the magnitude of magnetic force is proportional to (i) $|\nabla(|\boldsymbol{B}|)|$ or (ii)$\left|\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\mathbf{B}\right|$. In this paper, we make an effort to clarify the situation by demonstrating a very simple derivation for the earlier expression and proving that the latter expression does not hold in general.

Method: The magnetic forces on ferrous objects in a background magnetic field can be calculated in two ways defined below [3]:

$$\mathbf{F}=\nabla(\mathbf{m}\cdot\mathbf{B})\quad(1)\quad\text{and}\quad\mathbf{F}=(\mathbf{m}\cdot\nabla)\mathbf{B}\quad(2)$$

where $\mathbf{F}$ is magnetic force, $\mathbf{m}$ is the magnetic moment of the ferrous object and $\mathbf{B}$ is magnetic flux density of MRI magnets. It can be shown these two expressions are equivalent when $\mathbf{m}$ is not spatially dependent [3]. Below we first derive proposition (i) from force definition (1). Since $\mathbf{m}$ is not spatially dependent and can be expressed as $\chi_v\mathbf{B}$ (χ_v is the magnetic susceptibility), we have:

$$\mathbf{F}=\nabla(|\mathbf{m}||\mathbf{B}|\cos\theta)=|\boldsymbol{m}\cos\theta|\nabla(|\boldsymbol{B}|)\qquad(3)$$

So it is clear that $|\boldsymbol{F}|\propto|\nabla(|\boldsymbol{B}|)|$ if the object is saturated and $|\boldsymbol{F}|\propto|\mathbf{B}||\nabla(|\boldsymbol{B}|)|$ if the object is not saturated.

Next we use force definition (2) to present the proof that proposition (ii), where $|\mathbf{F}|\propto\left|\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\mathbf{B}\right|$, is not true in the general case. Since the object is small (big ferrous objects cannot be brought close to MRI), it can be assumed m_x, m_y and m_z are approximately constant within the object and they are related by the following equations:

$$m_y=m_x+\delta\quad(4)\quad\text{and}\quad m_z=m_y+\gamma\quad(5)$$

where δ and γ are real numbers. Take (4) & (5) into equation (2), we arrive at:

$$\mathbf{F}=m_x\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\mathbf{B}+\left(\delta\frac{\partial}{\partial y}+\gamma\frac{\partial}{\partial z}\right)\mathbf{B}\quad(6)$$

Let $\mathbf{F}_1=m_x\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\mathbf{B}$ and $\mathbf{F}_2=\left(\delta\frac{\partial}{\partial y}+\gamma\frac{\partial}{\partial z}\right)\mathbf{B}$, so $\mathbf{F}=\mathbf{F}_1+\mathbf{F}_2$. It is clear that $|\boldsymbol{F}_1|\propto\left|\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\mathbf{B}\right|$ but for $|\mathbf{F}|\propto\left|\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\mathbf{B}\right|$ to be true, $\mathbf{F}_2$ must be equal to zero. Now let's examine the conditions that constrain $\mathbf{F}_2$ to be zero. There are only five conditions that allow $\mathbf{F}_2$ to be zero. Below we show that each condition is highly specialized and does not hold for the field around an MRI magnet.

Condition 1: if $\delta=\gamma=0$, then $\mathbf{F}_2=0$. This means $m_x=m_y=m_z$. Since $\propto\chi_v\mathbf{B}$, and B_x, B_y and B_z vary widely around the MRI magnet, this condition cannot be satisfied.

Condition 2: if $\frac{\partial\mathbf{B}}{\partial y}=\frac{\partial\mathbf{B}}{\partial z}=0$, then $\mathbf{F}_2=0$. This leads to $\mathbf{F}=\mathbf{F}_1=m_x\left|\frac{\partial\mathbf{B}}{\partial x}\right|$. However, considering the divergence free and curl free property of $\mathbf{B}$, it requires $\frac{\partial\mathbf{B}}{\partial x}=0$. So the field is homogenous and force is zero. This condition obviously does not apply around an MRI magnet.

Condition 3: if $\delta=0,\gamma\neq0,\left|\frac{\partial\mathbf{B}}{\partial y}\right|\neq0,\left|\frac{\partial\mathbf{B}}{\partial z}\right|=0$, then $\mathbf{F}_2=0$. This condition combined with curl free property of $\mathbf{B}$ asks for $B_z=constant$. None of the B-component is constant around an MRI magnet, this condition cannot be met.

Condition 4: if $\delta\neq0,\gamma=0,\left|\frac{\partial\mathbf{B}}{\partial y}\right|=0,\left|\frac{\partial\mathbf{B}}{\partial z}\right|\neq0$, then $\mathbf{F}_2=0$. Same reasoning as condition 3 applies.

Condition 5: if $\delta\frac{\partial\mathbf{B}}{\partial y}=-\gamma\frac{\partial\mathbf{B}}{\partial z}$, then $\mathbf{F}_2=0$. This condition demands the partial derivatives with respect to *x* and *y* are proportional to each other with constant ratio defined by δ and γ across all space. The magnetic fields generated by MRI magnets are governed by Laplace's equation and the expansion of Green's function solution gives $1/r^n$ dependence for external field, and r^n dependence for internal field. In any region of the space around MRI magnet, the partial derivatives of flux density cannot hold constant ratio between them. The derivation of multipole expansion of localized current source distribution [4] clearly reveals this point and will not be repeated here.

Conclusion: In summary, we have considered all possible conditions under which $\mathbf{F}_2=0$ can be true. These are all very special cases which are not representative for the general situation where the magnetic force around MRI magnets is being evaluated. Therefore, $|\nabla(|\boldsymbol{B}|)|$, not $\left|\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\mathbf{B}\right|$, should be used for magnetic force estimation around MRI magnets.

Reference:

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