Incorporation of Rician Noise in the Analysis of Biexponential Transverse Relaxation in Cartilage using a Multiple Gradient Echo Sequence at 3T and 7T

Mustapha Bouhrara[1], David A Reiter[1], Hasan Celik[1], Jean-Marie Bonny[2], Vanessa Lukas[1], Kenneth W Fishbein[1], and Richard G Spencer[1]
[1]*NIA, NIH, Baltimore, Maryland, United States,* [2]*QuaPa, INRA, Saint Genès Champanelle, France*

Purpose: The noise distribution of magnitude MR data obtained through quadrature detection exhibits a Rician probability distribution. This approaches the Gaussian distribution only in the regime of high signal-to-noise ratio (SNR), so that the distinction is especially important in settings of limited SNR, including long echo-time images. Previous investigations have evaluated the performance of different analytic methods for extracting relaxation times (*e.g.* T_1, T_2) assuming monoexponential signal decay [1-2]. However, biexponential analysis of relaxation in tissue, including cartilage, is of increasing interest [3]. We therefore analyzed biexponential T_2^* decay in the presence of Rician noise and assessed the accuracy and precision of four different methods to determine component fractions and short, $T_{2,s}^*$, and long, $T_{2,l}^*$, spin-spin relaxation times. Our secondary goal was to investigate the use of the multi gradient echo sequence (MGES) to map biexponential T_2^* components in cartilage. All the following studies were conducted under both pre-clinical imaging conditions (PCI) at 7T and clinical imaging conditions (CI) at 3T.

Theory: Given Gaussian noise of equal variance, σ^2, in the real and imaginary channels of complex-valued MRI data, the expectation value of the measured magnitude signal, S_M, is given by $E_{Rice}[S_M] = \sigma\sqrt{\pi/2}\, e^{-\alpha}\big((1+2\alpha)I_0(\alpha) + 2\alpha I_1(\alpha)\big)$, were, $\alpha = (A(\mathbf{\theta}, TE)/2\sigma)^2$ and I_i is the modified i^{th} order Bessel function of the first kind. Here, $A(\mathbf{\theta}, TE) = \rho\left(F_s exp(-TE/T_{2,s}^*) + (1-F_s) exp(-TE/T_{2,l}^*)\right)$ is the magnitude of the underlying noise-free signal as a function of echo time *TE*, ρ, the signal amplitude at *TE* = 0, and parameters $\mathbf{\theta} = [\rho\ F_s\ T_{2,s}^*\ T_{2,l}^*]$, where F_s is the fraction of the short T_2^* relaxation component. For $A(\mathbf{\theta}, TE) \gg \sigma$, the Rician PDF approximates a Gaussian PDF with expectation value given by $E_{Gauss}[S_M] = A(\mathbf{\theta}, TE)$.

Methods: *Methods comparison:* Four different biexponential fitting methods were compared, using Monte Carlo simulations (MCs), phantom measurements, and *ex-vivo* cartilage studies. The Rician method, M_R, consists of nonlinear least-squares (NLLS) minimization of $S_M - E_{Rice}[S_M]$. The traditional method, M_T, consists of NLLS minimization of $S_M - E_{Gauss}[S_M]$. The McGibney method [4], M_{Mc}, consists of NLLS minimization of $S_M^2 - E_{Gauss}[S_M^2]$, where $E_{Gauss}[S_M^2] = A^2(\mathbf{\theta}, TE) + 2\sigma^2$. The Gudbjartsson method [5], M_G, consists of the NLLS minimization of $\sqrt{|S_M^2 - \sigma^2|} - E_{Gauss}[S_M]$. To eliminate one degree of freedom in the NLLS fit, σ was considered known in the MCs and calculated from background regions in the images where $A(\mathbf{\theta}, TE) = 0$ in phantom and *ex-vivo* studies [6]. We used a curve peeling algorithm to select the initial estimates for all analyses. *Bias and dispersion calculations*: The performance of each method was evaluated as a function of SNR. The comparison consisted of calculating the relative bias, a measure of accuracy, defined as $100 * |\theta_i - \hat{\theta}_i|/\theta_i$, where θ_i and $\hat{\theta}_i$ were true and estimated parameter values, respectively, and the relative dispersion, a measure of precision, defined as the relative standard deviation, $100 * \mathrm{SD}(\hat{\theta}_i)/\theta_i$. The later was compared to the minimal relative dispersion given by Cramér-Rao lower bound (CRLB) calculations. *MCs*: Simulation results were obtained for different values of F_s and the ratio $T_{2,l}^*/T_{2,s}^*$ *(Table 1)*. Rician noise was added to the synthetic images (70 x 70 pixels) prior to analysis. *Phantom study*: Biexponential analysis was performed on data consisting of 32 T_2^*-weighted images with *TE* increasing linearly. Images were obtained at 7T and 3T using the MGES, from two tubes of differently doped water. The number of signal averages was varied to produce images with different SNR. *Ex-vivo application*: Bovine nasal cartilage (BNC) was imaged at 7T and 3T using MGES. The sample was immersed in Fluorinert FC-77 to maintain sample hydration and to avoid susceptibility artifacts.

Results: At high SNR, MCS (Fig.1a) and phantom measurements (Fig.1b) show that all methods performed well. At lower SNR, the method explicitly incorporating the analytic form of the Rician noise, M_R, was the most accurate and approaches the CRLB bound (Fig.1 and Table 1). M_G performed as well in many cases, consistent with previous results for the monoexponential [1,5]. The M_{Mc} method showed relatively large dispersion, and minimal bias improvement over M_T, consistent with previous analyses [1,5] in which it was attributed to the fact that the noise distribution in power images approximates neither the Rician nor the Gaussian distribution at low SNR. In general, both accuracy and precision decrease with a decreasing ratio $T_{2,l}^*/T_{2,s}^*$, while bias and dispersion increase with increasing disparity between component sizes (Table 1). *Ex-vivo* experiments on phantoms and BNC were consistent with MCs and CRLB results.

Conclusions: Explicit incorporation of Rician noise greatly improves accuracy and precision in the analysis of biexponential transverse decay data. Combining the MGES for rapid data acquisition with the M_R method for voxel-by-voxel data fitting may represent a promising approach to bicomponent mapping in human articular cartilage.

References: *[1] Karlsen OT et al. MRM 1999;41:614-623. [2] Raya JG et al. MRM 2010;63:181-193. [3] Pauli C et al. Radiology 2012;264:484-493. [4] McGibney G et al. Med Phys 1993;20:1077-1078. [5] Gudbjartsson H et al. MRM 1995;34:910-914. [6] Sijbers J et al. MRI 1998;16:87-90.*

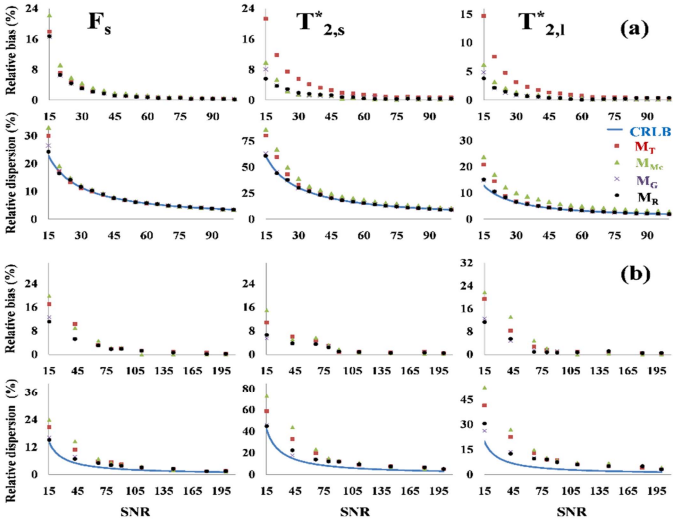

Fig.1. Relative bias and dispersion of F_s, $T_{2,s}^*$ and $T_{2,l}^*$ as a function of SNR using the fitting methods M_T, M_{Mc}, M_G and M_R. a) Simulation results under clinical conditions (CI) and b) phantom results under preclinical conditions (PCI). Input parameters for CI were $F_s = 0.5$, $T_{2,s}^* = 3.3$ ms, $T_{2,l}^* = 38$ ms, TE_1= 2.6 ms and ΔTE=2.4 ms, and for PCI were $F_s = 0.5$, $T_{2,s}^* = 3.6$ ms, $T_{2,l}^* = 41$ ms, TE_1 = 0.9 ms and ΔTE = 1.7 ms.

	M_T		M_{Mc}		M_G		M_R	
	Accu (%)	Preci (%)	Accu (%)	Preci (%)	Accu (%)	Preci (%)	Accu (%)	Preci (%)
For F_s = 0.3, $T_{2,s}^*$ = 4 ms and $T_{2,l}^*$ = 40 ms								
F_s	44	58	56	71	38	50	37	51
$T_{2,s}^*$	24	80	18	93	17	74	14	74
$T_{2,l}^*$	14	20	11	23	5	16	5	15
For F_s = 0.7, $T_{2,s}^*$ = 4 ms and $T_{2,l}^*$ = 40 ms								
F_s	81	74	78	81	69	68	58	61
$T_{2,s}^*$	47	98	12	114	18	91	6	89
$T_{2,l}^*$	57	92	10	31	22	38	8	23
For F_s = 0.3, $T_{2,s}^*$ = 4 ms and $T_{2,l}^*$ = 20 ms								
F_s	5	6	6	13	3	7	2	7
$T_{2,s}^*$	8	35	9	44	3	30	1	28
$T_{2,l}^*$	35	42	15	58	12	31	8	31

Table 1. Simulation results for relative accuracy and precision of estimates of F_s, $T_{2,s}^*$ and $T_{2,l}^*$ at SNR = 15 using the M_T, M_{Mc}, M_G and M_R methods for three different combination of input parameters F_s, $T_{2,s}^*$ and $T_{2,l}^*$. Here TE_1 = 2 ms and ΔTE = 2 ms.