A Novel Approach for Statistical Estimation of HARDI Diffusion Parameters from Rician and Non-Central Chi Magnitude Images

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Introduction: The statistics of noisy MRI magnitude and root sum-of-squares (SoS) images are accurately modeled by the Rician and non-central chi (NCC) probability distributions, respectively [1]. In diffusion MRI (dMRI), the noise bias introduced by these distributions leads to distortion of estimated quantitative diffusion parameters [2]. One approach to addressing this noise bias is to denoise the data prior to parameter estimation [3,4]. Another strategy is to apply maximum likelihood (ML) estimation methods to quantify diffusion, leveraging the known characteristics of the Rician/NCC distribution [5]. Due to computational complexity, the ML strategy has been primarily used in dMRI for simple DTI fitting with Rician noise, while use of ML estimation with more advanced dMRI models and/or NCC noise is much less common. However, recent work has demonstrated an efficient algorithmic majorize-minimize (MM) framework for solving ML estimation problems involving Rician and NCC distributions [6]. In this work, we adapt the MM framework from [6] to the context of estimating high angular resolution diffusion imaging (HARDI) parameters from Rician and NCC data.

Theory: Many HARDI estimation methods start by estimating the spherical harmonic (SH) representation of the measured diffusion profile [7], which is achieved by finding the SH coefficient vector $\mathbf{c}$ that minimizes $\|\mathbf{Bc}-\mathbf{y}\|_{\ell_2}^2+\lambda\|\mathbf{Lc}\|_{\ell_2}^2$, where $\mathbf{y}$ is the measured diffusion data, $\mathbf{B}$ is a matrix of sampled SH basis functions, $\mathbf{L}$ is a Laplace-Beltrami regularization matrix, and λ is a regularization parameter. While this problem is easy to solve, the solutions can be highly biased in the presence of non-Gaussian noise. In this work, we instead find $\mathbf{c}$ that optimizes the regularized ML functional given by: $\mathcal{L}(\mathbf{y}|\mathbf{c})+\lambda\|\mathbf{Lc}\|_{\ell_2}^2$, where $\mathcal{L}(\mathbf{y}|\mathbf{c})$ is the negative log-likelihood for Rician/NCC data [1]. While this optimization problem is nonconvex, the MM framework from [6] allows us to solve the difficult ML optimization by iteratively solving simpler least-squares cost functions. In particular, at the ith iteration, $\mathbf{c}^{(i)}$ is chosen to minimize $\|\mathbf{Bc}-\tilde{\mathbf{y}}\|_{\ell_2}^2+\lambda\|\mathbf{Lc}\|_{\ell_2}^2$ subject to $\mathbf{Bc}\geq\mathbf{0}$, where $[\tilde{\mathbf{y}}]_i=[\mathbf{y}]_i\mathrm{I}_n\left(\frac{[\mathbf{Bc}^{(i-1)}]_i[\mathbf{y}]_i}{\sigma^2}\right)/\mathrm{I}_{n-1}\left(\frac{[\mathbf{Bc}^{(i-1)}]_i[\mathbf{y}]_i}{\sigma^2}\right)$, σ^2 is the Gaussian noise variance, and $\mathrm{I}_n(\cdot)$ is the n^{th}-order modified Bessel function of the first kind. This iteration monotonically decreases the ML cost function [6].

Methods: The proposed MM approach was evaluated with simulated and real data. Simulated data was generated by sampling the analytic *q*-space signal expression for a mixture of two differently-oriented diffusion tensor models with equal eigenvalues and volume fractions, and adding noise. I*n vivo* diffusion data was also acquired using a 32-channel array coil. The proposed approach was compared to two alternative processing strategies: the Gaussian Approximation (GA) estimates the diffusion profile by minimizing $\|\mathbf{Bc}-\mathbf{y}\|_{\ell_2}^2+\lambda\|\mathbf{Lc}\|_{\ell_2}^2$ [7]. The Squaring Transform (ST) approach squares the noisy magnitude images, subtracts the noise bias, and takes the root of the result [8]. The GA approach is then applied to the "debiased" data.

Results: Fig. 1 compares the true diffusion profile (two tensors at $90°$) against estimated diffusion profiles from Rician data by GA, ST, and the proposed MM method. Qualitatively, ST and the proposed MM method outperform GA at all SNRs. At high SNR (≥8), ST and MM perform equally well, while the proposed MM approach is superior at lower SNRs. This is expected because the proposed MM method models noise more accurately. Fig. 2 shows several quantitative parameters derived from simulated data with SNR=3: absolute error in the diffusion profile (Err), mean diffusivity (MD), and generalized fractional anisotropy (GFA). The results indicate that the proposed method has more precise estimates of MD and GFA, and lower Err than ST or GA. However, the use of ML estimation in the proposed method can increase the variance of the estimated parameters compared to GA or ST, which is consistent with previous observations involving the Rician distribution and the DTI model [5]. Fig. 3 shows the result of applying NCC estimation to 32 channel SoS real diffusion data at increasing b-values (decreasing SNR). It can be seen that the proposed method is able to substantially reduce noise in the images, even at high b-values where the SNR is very low. Traditionally, denoising of diffusion images is achieved by spatial smoothing [3,4]. These results demonstrate that high-quality voxel-by-voxel denoising is also possible using Laplace-Beltrami smoothing and appropriate noise modeling.

Conclusion: We have proposed and investigated a novel technique for ML estimation of HARDI diffusion parameters, and demonstrated that using accurate Rician/NCC modeling of the statistical data distribution can substantially improve estimation results relative to conventional approaches. While we focused on voxel-by-voxel estimation of HARDI parameters, the approach is easily generalized to other diffusion models, and/or to include additional constraints (like the spatial smoothness of diffusion parameters).

References : [1] O. Dietrich, *MRI* 26,2008. [2] D. Jones, *MRM* 52, 2004. [3] V. Brion, *MRI* 31, 2013. [4] F. Lam, *MRM* Early View, 2013. [5] J. Andersson, *NeuroImage* 42, 2008 [6] D. Varadarajan, *IEEE ISBI* 2013. [7] M. Descoteaux, *MRM* 58, 2007. [8] H. Gudbjartsson, *MRM* 34, 1995.

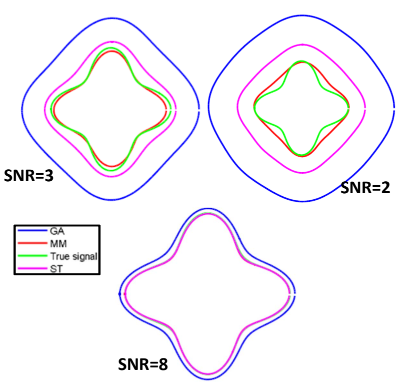

Fig. 1. Estimated diffusion profiles from simulated data at different. The proposed MM method matches the true signal, while GA and ST show significant bias at low SNR.

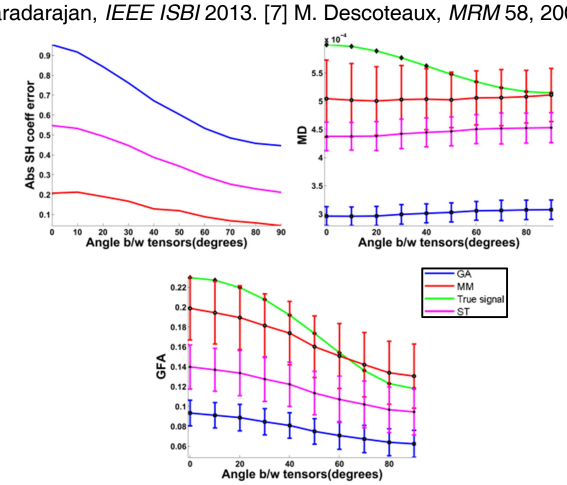

Fig. 2. Err, MD, and GFA plotted as a function of the angle between two simulated diffusion tensors.

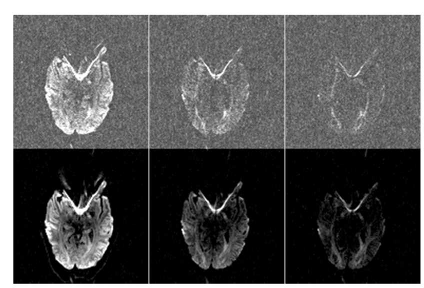

Fig 3. HARDI denoising based on voxel-by-voxel NCC modeling with Laplace-Beltrami SH regularization. Left to right: images with *b*-values of 1000, 2000, and 3000 s/mm^2. Top: original images. Bottom: results of regularized NCC estimation of the SH coefficients.