Is the Gaussian Phase Approximation Valid for the Blood Compartment in IVIM Imaging?

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Target audience: The work at hand is of interest to researchers in the field of diffusion MRI aiming to quantify intravoxel incoherent motion.

Purpose: To investigate the validity of using the Gaussian phase approximation (GPA) to calculate the signal attenuation of the perfusion compartment in case of intravoxel incoherent motion (IVIM).

Methods: Following Le Bihan *et al.*[1], incoherent motion can be described by its characteristic velocity v and the timescale τ needed to travel a capillary segment (Fig. 1a). The phase □ acquired by a moving particle during application of a diffusion gradient profile of total duration T scales as $\phi(b,T,\tau,v) = v\sqrt{bT}\varphi\left(\frac{T}{\tau}\right)$, motivating the introduction of the normalized phase φ. The distribution $\rho(\varphi,N)$ for a given diffusion profile, which is needed to calculate the signal attenuation F of the incoherent motion compartment, is only dependent on the ratio $N=\frac{T}{\tau}$. Normalized phase distributions for the diffusion profiles depicted in Fig. 1b were obtained by simulating $6.4 \cdot 10^6$ particle paths for a given N and are shown in Fig. 1c and Fig. 1d.

The signal attenuation calculated from normalized phase distributions $F(b,T,\tau,v) = \left|\int_{-\infty}^{\infty} \rho\left(\varphi,\frac{T}{\tau}\right) e^{iv\sqrt{bT}\varphi} \mathrm{d}\varphi\right|$ is compared to the GPA given by $F(b,T,\tau,v) = \exp\left[-bTv^2 \left|\int_{-\infty}^{\infty} \rho\left(\varphi,\frac{T}{\tau}\right)\frac{\varphi^2}{2}\mathrm{d}\varphi\right|\right]$ which is equivalent to the autocorrelation method proposed by Kennan *et al.*[2]

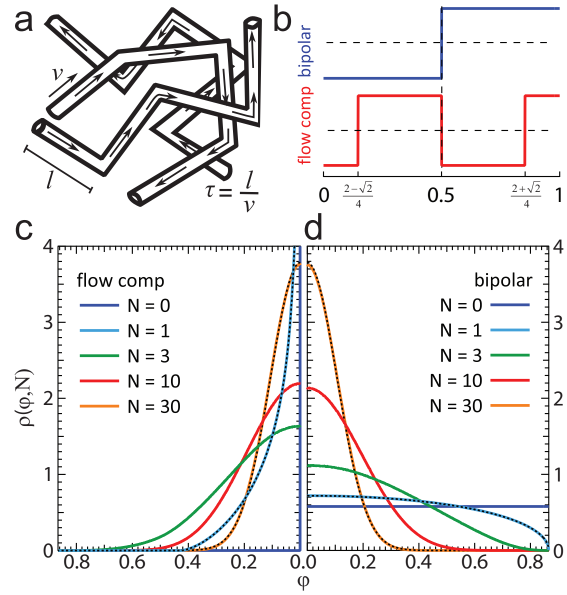

Fig. 1a: incoherent motion in a capillary network, b: normalized diffusion gradient profiles, c+d: normalized phase distributions for the profiles in b. For large N a Gaussian is approached (dotted line for N=30), for N=1 analytic solutions can be found (dotted lines).

Results: While, for large N, phase distributions converge to a Gaussian and the signal attenuation F becomes independent of the applied gradient profile, the situation is different for small N. In Fig. 2 signal attenuation curves for the case N=1 and $D^* = \frac{v^2\tau}{6} = 187\,\frac{\mu\mathrm{m}^2}{\mathrm{ms}}$ are depicted. For a single velocity (Fig. 2a) the GPA can descibe the bipolar data quite well and the flow comp data at least at b < 100 s/mm². If a parabolic velocity profile is however included into the normalized phase distributions, the GPA breaks down completely.

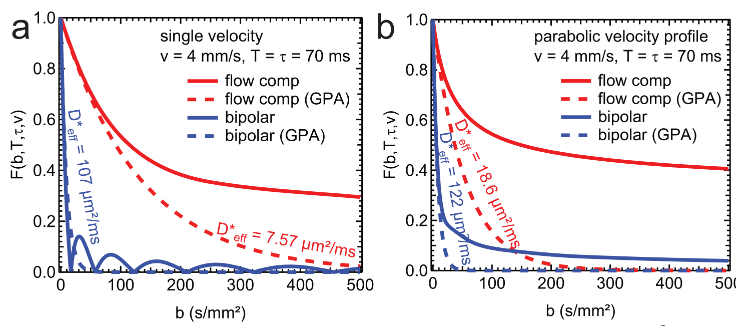

Fig. 2a+b: IVIM signal attenuation for N=1 and D^*=187 $\frac{\mu\mathrm{m}^2}{\mathrm{ms}}$ differs for GPA and full phase information, in particular for the flow comp. profile. This effect is more pronounced, when a parabolic velocity distribution (b) is assumed instead of a single velocity (a).

Discussion: The autocorrelation method[2] by Kennan *et al.* is a versatile formalism to calculate the IVIM signal for arbitrary gradient profiles and estimate the effective initial signal decay at small b-values correctly. It is governed by D^*_{eff}, which is related to the pseudo-diffusion coefficient $D^* = \frac{v^2\tau}{6}$, as defined[1] by Le Bihan, by $D^*_{\mathrm{eff}} = D^* \cdot 3N\langle\varphi^2\rangle$. For $\tau \sim T$ the GPA breaks down at larger b-values or in case of a larger number of velocity components such that the IVIM signal cannot be described adequately by a pseudo-diffusion coefficient anymore. While this might not necessarily be the case for all IVIM applications, different signal attenuation curves for flow compensated and bipolar gradients were reported[3] in liver and pancreas strongly indicating that the GPA doesn't hold for those organs.

Conclusions: To quantify intravoxel incoherent motion, it is advisable to ensure that the GPA is valid before performing a biexponential fit, e.g. by comparing the bipolar to flow compensated diffusion data. Even if the GPA holds, it should be kept in mind, that D^*_{eff} obtained by a biexponential fit does not necessarily correspond to the original D^* which is directly related to τ and v. Those parameters can be assessed by flow compensated diffusion gradients[3] which potentially allows one to quantify changes in microvasculature.

References: [1] D. Le Bihan *et al.*, *Radiology* **168**, p.497 (1988) [2] R. P. Kennan *et al.*, Med. Phys. 21, p.539 (1994)
[3] A. Wetscherek *et al.*, *Proc ISMRM 20th Annual Meeting*, p.2012 (2012)