# Integrating Principal Component Analysis and Dictionary Learning with Coherence Constraint for Fast $T_{1\rho}$ Mapping

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**INTRODUCTION** Long scanning time greatly hinders the widespread application of spin-lattice relaxation in rotating frame ($T_{1\rho}$) in clinics. Compressed sensing (CS), a novel theory used for fast MR imaging with higher degree of undersampling may have potential to accelerate $T_{1\rho}$ imaging. A fundamental question in the successful applications of CS is the choice of sparsifying transform. CS-MRI with fixed, global sparsifying transform such as wavelet or finite-difference is usually limited to low acceleration factors, since a given basis might not be universally optimal for all images. To address this issue, adaptive transform such as learned dictionary has been introduced to reconstruct MR images from highly undersampled k-space data[1, 2]. Most recently, the fixed and adaptive transform were combined to further promote the sparsity of the signal and this strategy has shown promising reconstruction[3]. However, the bases of the learned dictionary could be coherent with those from the fixed transform, which may alleviate the representation ability of the dictionary. In this paper, we propose a novel method for accelerating $T_{1\rho}$ imaging. Specifically, a new constraint that characterizes the coherence between the learned dictionary and fixed transform was added to the model proposed in Ref.3. The improvement of the proposed method over the one without coherence constraint and the conventional dictionary-learning based method is demonstrated using *in vivo* brain $T_{1\rho}$ experiments.

**THEORY AND METHOD** In our method, PCA is applied first along the parameter direction, and then the dictionary learning (DL) technique is utilized to reconstruct the sparse PC coefficients under the coherence and data consistency constraints. The resulting $T_{1\rho}$-weighted image series are obtained with the inverse PCA transform. The reconstruction problem can be formulated as

$$\min_{\mathbf{x},\mathbf{D},\alpha_n}\left\{\sum_n\left\|\mathbf{D}\alpha_n - R_n(\mathbf{Bx})\right\|_2^2 + \nu\left\|F_p\mathbf{x}-\mathbf{y}\right\|_2^2 + \zeta\left\|\mathbf{B}^T\mathbf{D}\right\|_F^2\right\}$$
$$s.t. \quad \left\|\alpha_n\right\|_0 \le T_0, \forall n$$

where $\mathbf{x}$ is a $N$ ( voxel number )$\times L$ ( echo number ) matrix, representing the image series to be reconstructed. $F_p$ is the undersampled Fourier operator. $\boldsymbol{y}$ is the measured k-space data. $\mathbf{B}$ is the matrix of the PC vectors, with the corresponding singular values ordered from the largest to the smallest. It is generated by using a training data set from $\mathbf{x}$ with the SVD algorithm and updated iteratively in reconstruction. $R_n$ represents the operator that extracts the PC coefficients at the same spatial location. $\mathbf{D}$ is the patch based dictionary composed by a set of atoms. $\alpha_n$ is the sparse representation of patch $n$ with respect to $\mathbf{D}$. $T_0$ is the required sparsity level. The regularization parameter $\nu$ is set as $\nu = (\lambda/\sigma)$ , where $\sigma$ is the standard deviation of the measurement noise and $\lambda$ is a positive constant. $\zeta$ is the weight of the coherence regularization. As we can see the first term in the cost function captures sparse prior of the PC coefficients of the signal with respect to the dictionary. The second term enforces data fidelity in k-space and the last term guarantees the incoherence between the bases of two transforms. Experiment The experiments were performed on a Philips Achieva 3T clinical scanner. $T_{1\rho}$-weighted images were fully acquired using a rotary-echo spin-lock pulse embedded turbo spin-echo (TSE) sequence with an 8-channel head array coil. Imaging parameters were: TE/TR=7/3000 msec, excitation pulse flip angle=90°, refocusing pulse flip angle=180°, echo train length ETL=16, pixel size=0.6×0.6mm$^2$, slice thickness=5mm and TSLs = 1, 20, 40, 60 and 80msec. $T_{1\rho}$ map was estimated by the least square fitting of the reconstructed $T_{1\rho}$ weighted images according to the exponential model.

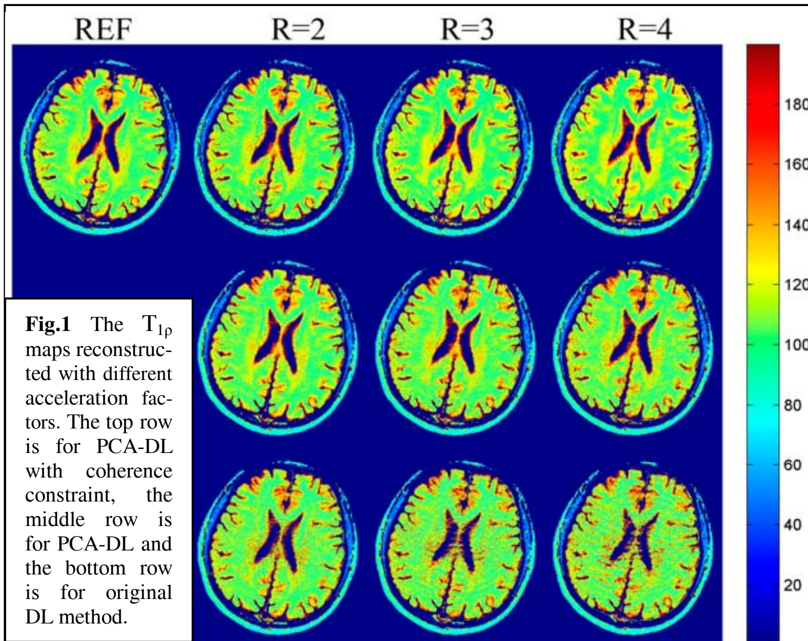


**Fig.1** The $T_{1\rho}$ maps reconstructed with different acceleration factors. The top row is for PCA-DL with coherence constraint, the middle row is for PCA-DL and the bottom row is for original DL method.

| | PCA-DL-Coherence | PCA-DL | DL |
|---|---|---|---|
| R=2 | 0.0707 | 0.0757 | 0.1099 |
| R=3 | 0.0815 | 0.0989 | 0.1397 |
| R=4 | 0.0989 | 0.1159 | 0.1603 |

Tab.1 Quantified performance of different methods

**RESULTS AND DISCUSSION** The $T_{1\rho}$ maps derived using DL, PCA-DL and PCA-DL with coherence constraint with different acceleration factors (R) are given in Fig.1. The DL method exhibits aliasing artifacts when R is 3, and the artifacts become more conspicuous at R=4. With PCA transform being integrated with DL, the artifacts were significantly alleviated. But the $T_{1\rho}$ map using PCA-DL still shows slight artifacts when R reaches 4. When the coherence constraint was added into PCA-DL, there are only negligible artifacts shown in the estimated map with R=4. The quantified performance of three methods was demonstrated in terms of root-mean-squared errors (RMSE) from the reconstructed $T_{1\rho}$ maps (see Table 1). It can be seen that the proposed method exhibits smallest errors, which is consistent with the visual observation.

**CONCLUSION** We propose to integrate PCA and dictionary learning with coherence constraint for fast $T_{1\rho}$ mapping. Experimental results demonstrate that combining dictionary learning and PCA transform can improve the accuracy of estimated $T_{1\rho}$ map using the DL technique only. More promisingly, the introduction of coherence constraint further promotes the performance of parameter mapping.

**REFERENCES** [1] Ravishankar S *et al.* IEEE TMI, 30(5):1028-1041, 2011 [2]Velikina JV *et al. MRM*, 70:1263-1273,2013 [3] Liu Q *et al.* IEEE TIP, 22(12):4652-4663, 2013.

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