P-LORAKS: Low-rank modeling of local k-space neighborhoods with parallel imaging data

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Introduction: Parallel imaging and constrained image reconstruction are two popular approaches that enable sub-Nyquist MRI experiments. This work explores the combination of parallel imaging with low-rank matrix modeling of local k-space neighborhoods (LORAKS) [1,2]. LORAKS, a recent constrained image reconstruction framework originally designed for single-coil data, makes use of linear dependencies in k-space that are present for images that are support-limited and/or have smooth phase. It was shown that LORAKS imposes support and phase constraints in a fundamentally different way from previous constrained reconstruction methods, can yield substantial improvements in reconstruction quality, and is flexible enough to be used with calibrationless k-space trajectories [1,2].

Theory and Methods: The LORAKS framework is based on the fact that low-rank matrices with Hankel-like structure can be constructed from the fully-sampled k-space data of images with limited spatial support and/or slowly varying phase. Specifically, $\mathbf{C}$ is a LORAKS matrix formed from k-space samples such that, if $\mathbf{B}$ is a small $N \times N$ 2D k-space kernel and $\mathbf{b}$ is its vectorized representation, then $\mathbf{Cb}$ implements the 2D convolution of $\mathbf{B}$ with the k-space data. It has been shown that $\mathbf{C}$ has low-rank when the image has limited support [2]. $\mathbf{G}$ and $\mathbf{S}$ are similar k-space convolution LORAKS matrices that collect information from opposite sides of k-space (based on known k-space symmetry relationships), and are low-rank when the image has smooth phase [1,2]. This matrix representation is powerful because compressed sensing approaches exist for reconstructing low-rank matrices [3], and LORAKS-based reconstruction can yield better results than traditional sparsity-based compressed sensing [2].

In parallel imaging, we observe k-space data $\mathbf{d}_\ell$ simultaneously from L different receiver coils. The proposed P-LORAKS method extends LORAKS by building large matrices according to $\mathbf{C}_{tot} = [\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_L]$, $\mathbf{G}_{tot} = [\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_L]$, and $\mathbf{S}_{tot} = [\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_L]$, where $\mathbf{C}_\ell$, $\mathbf{G}_\ell$, and $\mathbf{S}_\ell$ are the LORAKS matrices for ℓth coil. It is observed that $\mathbf{C}_{tot}$, $\mathbf{G}_{tot}$, and $\mathbf{S}_{tot}$ will not only have nullspace vectors corresponding to the LORAKS constraints, but will also have nullspace vectors corresponding to any linear dependence relationships between the different coils. Note that widely-used methods like GRAPPA and SPIRiT depend on the existence of such relationships between the data from multiple coils [4,5]. Also note that P-LORAKS using the $\mathbf{C}_{tot}$ matrix is nearly identical to the SAKE method [6], though was derived in a different way. P-LORAKS based on $\mathbf{G}_{tot}$ and/or $\mathbf{S}_{tot}$ is distinct from previous methods.

P-LORAKS reconstructions from undersampled data are obtained by minimizing $\sum_{\ell=1}^{L} \|\mathbf{F}\mathbf{k}_\ell - \mathbf{d}_\ell\|_{\ell_2}^2 + \lambda_C J(\mathbf{C}_{tot}) + \lambda_G J(\mathbf{G}_{tot}) + \lambda_s J(\mathbf{S}_{tot})$ with respect to the unknown complete k-space vectors $\mathbf{k}_\ell$, $\ell = 1,2,\ldots,L$. In this expression, $\mathbf{F}$ is a Fourier-domain subsampling matrix, λ_C, λ_G, and λ_s are regularization parameters, and $J(\cdot)$ is a nonconvex penalty function that encourages the matrices to have low-rank structure based on prior knowledge of the approximate matrix rank [2]. Optimization is performed using an efficient majorize-minimize algorithm that alternates between computing truncated SVDs and solving simple least-squares problems [2].

Results: Figure 1 compares the normalized singular values of the LORAKS matrices to the normalized singular values of the P-LORAKS matrices for fully-sampled k-space data. The fact that the P-LORAKS singular values decay much more rapidly than the LORAKS singular values indicates that the P-LORAKS representation more effectively compresses this data, and should be more effective than the LORAKS representation for multi-coil MRI applications. A comparison between SPIRiT and P-LORAKS is shown for 8-channel data in Fig. 2. We observe that P-LORAKS can achieve accurate reconstructions from limited data, with advantages over SPIRiT. We also observe that P-LORAKS using phase information ($\mathbf{S}_{tot}$) is more effective than SAKE/P-LORAKS using support information ($\mathbf{C}_{tot}$), which is consistent with previous LORAKS results [2].

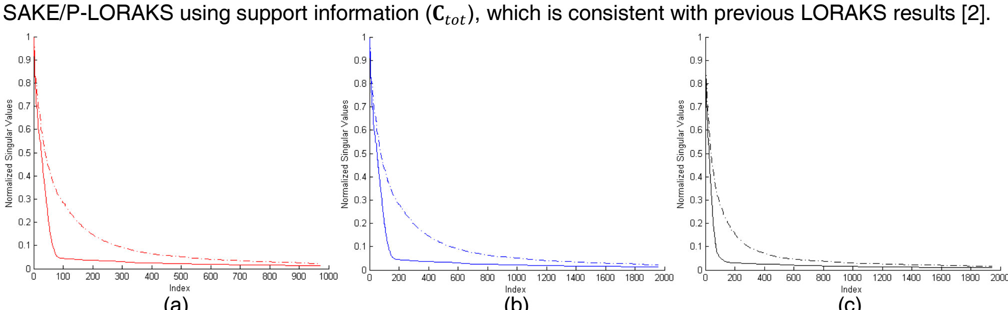

Fig. 1. Plots of the sorted singular values for P-LORAKS matrices (solid lines) and LORAKS matrices (dashed lines). For the same level of accuracy, the P-LORAKS representation leads to more parsimonious signal modeling than the LORAKS representation.
(a) $\mathbf{C}_{tot}$ versus $\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_L$.
(b) $\mathbf{G}_{tot}$ versus $\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_L$.
(c) $\mathbf{S}_{tot}$ versus $\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_L$.

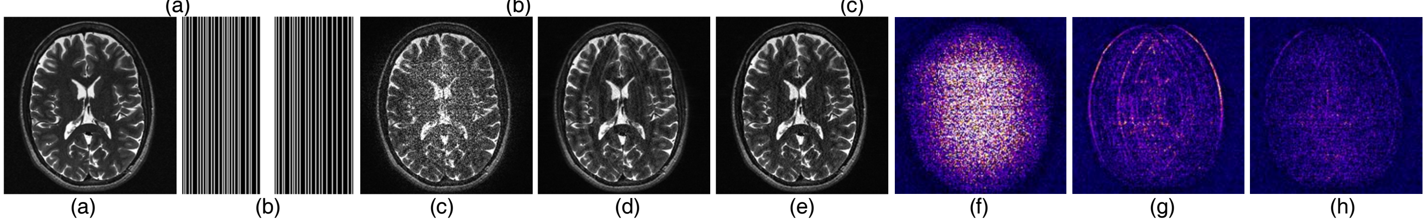

Fig. 2. Illustrative comparison between SPIRiT [5] and P-LORAKS. (a) Original image. (b) k-space sampling mask (3× acceleration). (c) SPIRiT reconstruction. (d) P-LORAKS reconstruction based only on the $\mathbf{C}_{tot}$ matrix (similar to SAKE [6]). (e) P-LORAKS reconstruction based only on the $\mathbf{S}_{tot}$ matrix. (f-h) Error images corresponding to (c-e).

Conclusions: P-LORAKS is a new kind of constrained parallel image reconstruction approach that merges LORAKS constraints with parallel imaging, and has certain advantages over existing methods. P-LORAKS can be used with calibrationless k-space trajectories (not shown due to space constraints), and is easily used in combination with other regularized reconstruction methods. We expect the approach to be useful in a range of different accelerated MRI experiments.

References: [1] J. Haldar, ISMRM Workshop on Data Sampling & Image Reconstruction, 2013. [2] J. Haldar, Submitted 2013. [3] B. Recht, SIAM Rev, 2010. [4] M. Griswold, Magn Reson Med, 2002. [5] M. Lustig, Magn Reson Med, 2010. [6] P. Shin, Magn Reson Med, Accepted 2013.