

# MRI Reconstruction by Learning the Dictionary of Spatial-frequency-Bands Correlation: A novel algorithm integratable with PI and CS to further push acceleration

Enhao Gong<sup>1</sup> and John M Pauly<sup>1</sup>

<sup>1</sup>Electrical Engineering, Stanford University, Stanford, CA, United States

**Target Audience:** MR researchers on reconstruction and clinical scientists for fast imaging

**Purpose:** Parallel Imaging (PI)<sup>1,2</sup> and Compressed Sensing (CS)<sup>3,4</sup> enable scan acceleration by exploiting data correlation among channels and data sparsity in transform domains respectively. However, the acceleration capability is limited by the channel-encoding capability<sup>5</sup>, noise increase and detail blurring. Dictionary Learning has been proposed as a sparsifying transform to improve MRI reconstruction<sup>6,7</sup>. In this work, we proposed a new algorithm using Dictionary Learning to further improve reconstruction of MRI by exploiting the correlation between image details in different spatial-frequency bands<sup>8</sup>. The results demonstrated advantages over existing PI-CS algorithms and the proposed algorithm can be integrated with PI and CS for further acceleration and improved reconstruction.

**Methods:** The main steps of the algorithm (Fig. 1) and implementation details are:

**1) Image patch pairs were randomly sampled from database (3 MR images).**

Shown in fig 1a and 1b, high resolution images were reconstructed from the full k-space with spatial-frequency bandwidth  $BW_H=1$  and low resolution images were reconstructed from the center  $\frac{1}{4}$  k-space with bandwidth  $BW_L=\frac{1}{4}$ . Image patch (5x5) pairs were randomly sampled at matched positions in the images reconstructed with different spatial-frequency bandwidths and formed sample matrix  $P=[P_L, P_H]$ .

**2) Dictionary Learning<sup>6-8</sup> was used to generate a dictionary** from the sample matrix.

The patch pair samples are assumed to be linear combinations of only a few basis patch pairs in the coding dictionary, which is a sparse representation of the correlation between image details in different spatial-frequency bands. Using the k-SVD algorithm<sup>9</sup>, the patch pair samples can be decomposed to form basis dictionary  $B=[B_L, B_H]$  by solving  $\min_{B,w} \|P - Bw\|_2 + \lambda \|w\|_1, s.t. \|B_i^T\| \leq 1$ , in which  $B$  contains the trained coding dictionary with norm constraints on each basis patch pair  $B_i$ , and  $w$  is the sparse coding coefficient. A dictionary containing 1000 basis patch pairs was trained from 40000 randomly sampled image patch pairs.

**3) An image can be reconstructed as a sparse representation of the dictionary.**

From  $k$ -space  $x$ , low spatial-frequency band  $D_L x$  and high spatial-frequency band  $D_H x$  were decomposed as a sparse representation of the basis in the learned dictionary.

$$SR(x) = \min_w SR(w, x) = \|F_P F^{-1} D_L x - B_L w\|_2^2 + \lambda_H \|F_P F^{-1} D_H x - B_H w\|_2^2 + \lambda_{L1} \|w\|_1$$

$D_{L,H}$  selects samples in corresponding spatial-frequency bands,  $w$  is the sparse coding coefficients forming patches from basis patches,  $F_P$  is the patch operator forming patches from image and  $F$  is Fourier transform.  $\lambda_H < 1$  was used, since we assumed low frequency information is more reliable. Accordingly, the full bandwidth  $k$ -space  $\hat{x}$  can be recovered from estimated coefficients:  $\hat{x} = D_H^{-1} F_P^T B_H w$ .

**4) Iterative image reconstruction (POCS) integrated with PI and CS.**

The proposed operation, enforcing sparse representation of the correlation in different spatial-frequency bands, was integrated with **Data Consistency** and optionally with **Parallel Imaging Self-Consistency** and **Sparsity Constraints**. Here,  $x$  is the  $k$ -space to be recovered.  $y$  is  $k$ -space measurement,  $D$  is undersampling operator,  $G$  is a SPIRiT kernel,  $\Psi$  is wavelet transform and parameters  $\lambda_*$  balance the constraints.

$$\min_x E(x) = \|Dx - y\|_2^2 + \lambda_{SR}(SR(x)) + \lambda_{PI} \|Gx - x\|_2^2 + \lambda_{CS} \|\Psi F^{-1} x\|_1$$

**Results:** A T1w and a T2w in-vivo multi-slice brain scan, acquired on a Philips 3T system (Philips Healthcare, Best, the Netherlands) with an 8-channel head coil, were used to validate the algorithm. One image from T1w scan was reconstructed using the dictionary trained from a different T1w image and 2 T2w images. Figure 2 shows the results corresponding to step 1 to step 3 of the proposed algorithm. Fully sampled single-channel data was synthesized and retrospectively undersampled with  $\frac{1}{4}$  center  $k$ -space sampled at reduction factor of 4. The reconstruction results were compared with CS<sup>3</sup>. Figure 3 shows the performance of the proposed algorithm (step 1-4) compared with PI-CS at reduction factor of 16. Data was retrospectively undersampled with 2D Variable Density Poisson Disk trajectory. SPIRiT and L<sub>1</sub>-SPIRiT<sup>4</sup> were implemented for comparison.

**Discussion:** The results demonstrated the advantages of the proposed algorithm. Moreover, the combination with PI and CS further empowers the undersampling for static MRI. In addition, our experiments showed the learning process can be conducted by using patches from a few existing MR images, which is applicable for clinical applications. The algorithm is expected to be improved with optimized undersampling trajectories, larger database, multiple dictionaries learned with more frequency bandwidths. Fast pulse sequences suffering resolution loss are also expected to be much improved using the proposed algorithm.

**Conclusion:** We proposed a Dictionary Learning based algorithm to improve undersampled MRI reconstruction by exploiting correlation of image details in different spatial-frequency bands. The algorithm is compatible with PI-CS to achieve better reconstruction and more acceleration.

**References:** [1] Pruessmann et.al MRM 1999 [2] Griswold, et.al. MRM 2002 [3] Lustig et. al. MRM 2007 [4] Lustig et. al. MRM 2010 [5] Zhang et. al. MRM 2012 [6] Ravishankar et. al. IEEE TMI 2011 [7] Opher et. al. IEEE STSP 2011 [7] Yang et. al. IEEE TIP 2012 [8] Aharon et.al. IEEE TSP 2006

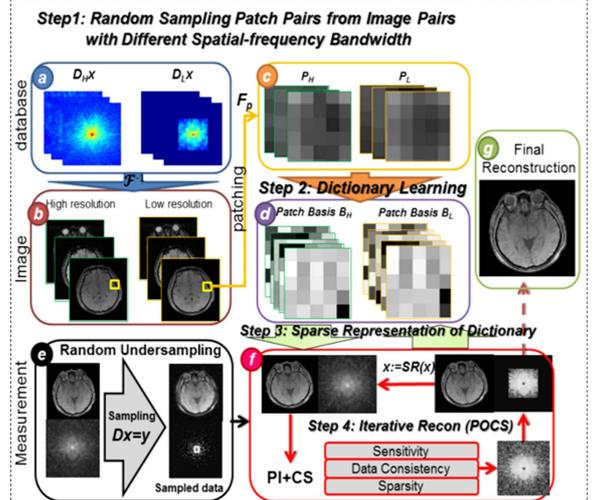


Fig 1. Flowchart of the algorithm: (a) k-space from database with different spatial-frequency bandwidth; (b) corresponding image from database with different resolution; (c) randomly sampled image patch pairs in step 1; (d) learned dictionary of patch pair in step 2; (e) new MR image data acquisition; (f) step 4 iterative reconstruction integrating Step 3 and PI+CS; (g) final reconstruction

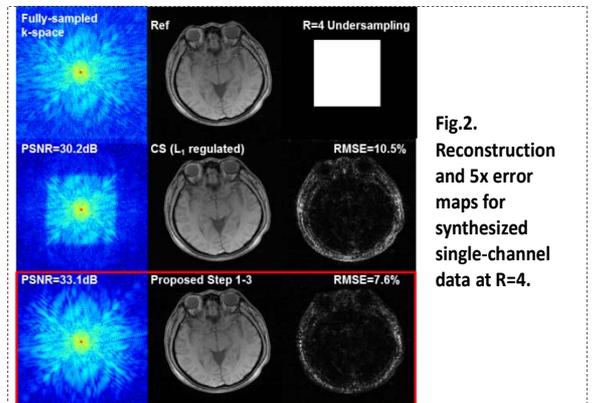


Fig.2. Reconstruction and 5x error maps for synthesized single-channel data at R=4.

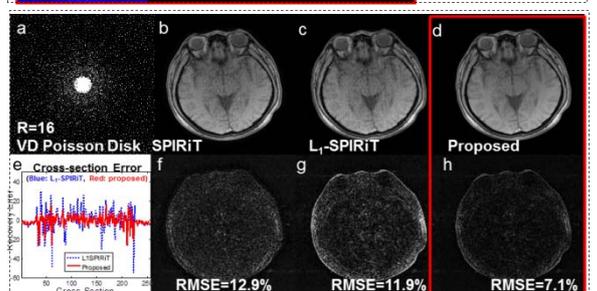


Fig.3. The proposed algorithm combining PI and CS achieved better reconstruction. (a) Sampling trajectory R=16, (b-d) Reconstruction of T1w brain image using different algorithms (e) Error of reconstruction along a cross-section, blue for L<sub>1</sub>SPIRiT and red for the proposed algorithm. (f) 5x error maps and normalized RMSE.