Calibration for Parallel MRI Using Robust Low-Rank Matrix Completion

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Target Audience: The target audience for this document includes those who are interested in MRI acceleration and motion compensation.

Introduction: The goal of this work is to develop a practical calibration method for parallel MRI which is robust against both under-sampling and corruption of the calibration data. It was demonstrated in [1, 2] that low-rank matrix completion can reconstruct non-uniformly under-sampled k-space without specific auto-calibration data (ACS). Furthermore the work in [3] demonstrated correction of k-space data corrupted by sparse motion. Here, we show a generalized formulation for motion-robust auto-calibration and reconstruction from under-sampled data that is incorporated into ESPIRiT [4]. The method is general and can incorporate navigation information when available. The feasibility of the method was demonstrated in simulation and *in-vivo* experiment.

Methods:

Low-rank matrix completion: Multi-channel k-space data is correlated and can be reformatted to form a low-rank block-Hankel matrix [1, 3]. Sparse errors due to motion corruption and missing k-space can be thought of as outliers that violate the low-rank property [5]. It is possible to restore both the missing and corrupted k-space by an outlier-robust low-rank matrix completion approach [6] of minimizing:

$$\hat{y} = \underset{\hat{y}}{\operatorname{argmin}} \|P\hat{y} - y\|_1, \;\; subject\ to{:}\, rank(H^{\dagger}\hat{y}) = k$$

Here, $\hat{y}$ is the estimated full k-space, y is the acquired data, P is the sampling operator, $H^{\dagger}$ is an operator that constructs a so called calibration matrix from the multi-channel k-space in which rows are sliding windows of the multi-channel data [1], and k is the apriori known rank of the calibration matrix. The l_1-norm in the objective is known to be robust to outliers. When navigator information is available, the robust method can be modified into a "weighted low-rank" algorithm [7, 8].

$$\hat{y} = \underset{\hat{y}}{\operatorname{argmin}} \|w(P\hat{y} - y)\|_2^2, \;\; subject\ to{:}\, rank(H^{\dagger}\hat{y}) = k$$

Experimentally, w can be calculated with a 3D navigator. These objective functions can be solved by iterative singular-value thresholding [2].

Combination with ESPIRiT: The central k-space, *i.e.* the calibration region for ESPIRiT, can be corrected by the proposed method. Then, this recovered auto-calibration region is used to compute eigenvector (sensitivity) maps. The full k-space can then be reconstructed using ESPIRiT. With navigator weights available, the weighted low-rank method is used to correct auto-calibration data and additionally, a weighted ESPIRiT is used to reduce corruptions in the final reconstructed image:

$$m = \underset{m}{\operatorname{argmin}} \|w(P\mathcal{F}Sm - y)\|_2^2 + \lambda\|\Psi m\|_1$$

Here, m is the reconstructed image, $\mathcal{F}$ is the Fourier transform, S are ESPIRiT eigenvector maps, and $\lambda\|\Psi m\|_1$ is a sparsity regularization.

Simulation: An 8-channel T1-weighted brain data acquired from 3D RF-spoiled FLASH (TR/TE=12.2/5.2ms, TI = 450ms, FA=20°, matrix size: 256×180×230) was artificially corrupted by random phase shifts in k-space and then retrospectively under-sampled by variable-density Poisson-disc sampling. 20% of the phase encoding lines were corrupted, followed by under-sampling without ACS region at a reduction factor of 5. After that, the corrupted k-space was corrected by the proposed methods. Weights for weighted methods were computed from the simulated motion. For comparison, images were also reconstructed by "conventional" ESPIRiT without any k-space correction.

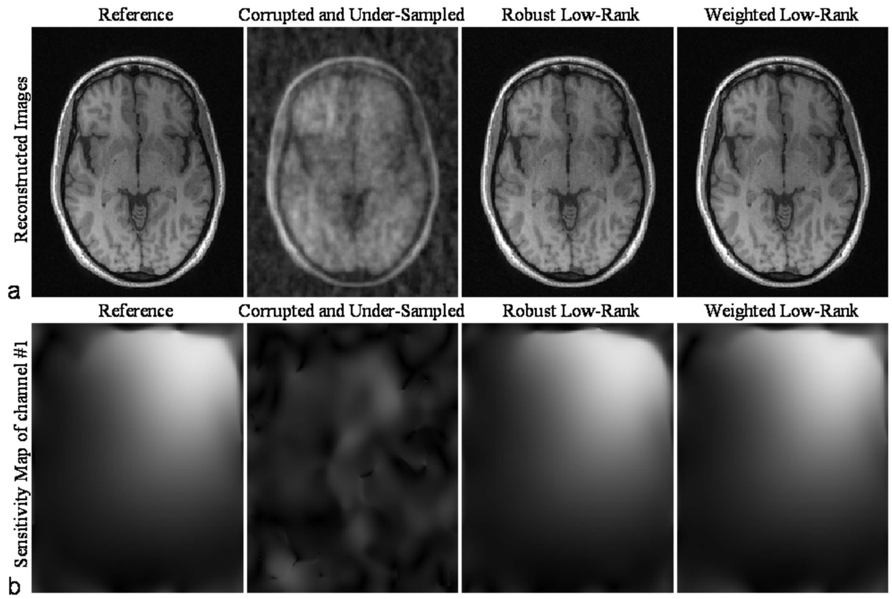

Fig 1. Simulation Results

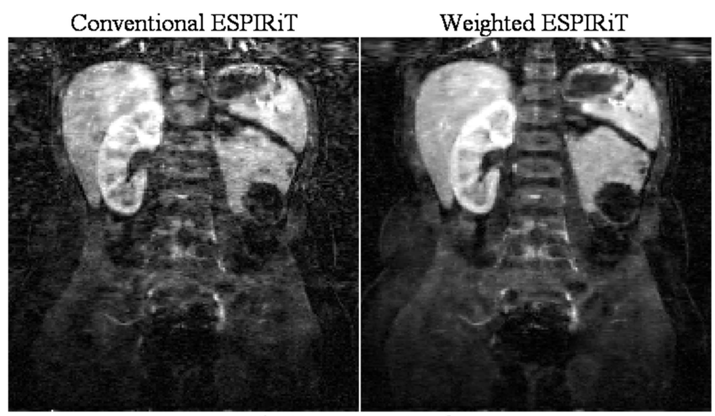

Fig. 2 *In-vivo* Results

In-vivo Experiment: Abdominal data from fat-suppressed 3D spoiled gradient echo sequence (TR/TE=3/1.2ms, FA=15°, matrix size: 192×180×68, 32 channels) was acquired during free-breathing using variable-density view-ordering (VDRad) [8]. Weights were obtained with a Butterfly navigator [8]. From this data set, a temporal phase with undersampling factor 8.4 was extracted and reconstructed with the proposed method. The reconstuction was conducted slice by slice after a 1D iFFT along frequency encoding direction. In order to reduce noise amplification, l_1-wavelet constraints were used in the reconstructions [4].

Results and discussion: Fig. 1a compares the simulation results of robust low-rank and weighted low-rank methods. Fig 1b shows the eigenvector maps computed from the calibration region via ESPIRiT with and without correction. These results demonstrate the efficiency of low-rank matrix completion on fixing corrupted and under-sampled data. Fig. 2 illustrates the results of *in-vivo* experiment. Weighted ESPIRiT significantly reduced corruption compared to not weighted ESPIRiT.

Conclusion: The proposed combination of ESPIRiT with low-rank matrix completion is robust against both under-sampling and corruption of the calibration region. Simulation and *in-vivo* experiment indicated the efficiency of the proposed method.

Reference: [1] Lustig, ISMRM, 2010:2870 [2] Shin, MRM, 2013, in press (available at http://www.eecs.berkeley.edu/~mlustig/preprints/SAKE.pdf) [3] Bi, ISMRM, 2013:2584 [4] Uecker, MRM, 2013, DOI: 10.1002/mrm.24751 [5] Huang, MRM, 2010;64(1):157-166 [6] Van Der Veen A-J, IEEE 1993;81(9):1277-1308 [7] Johnson, MRM, 2012;67(6):1600–1608 [8] Cheng, ISMRM, 2013:0312