Joint k-q Space Compressed Sensing for Accelerated Multi-Shell Acquisition and Reconstruction of the diffusion signal and Ensemble Average Propagator

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Introduction. High Angular Resolution Diffusion Imaging (HARDI) has been proposed to avoid the limitations of the conventional Diffusion Tensor Imaging (DTI) and to better explore white matter micro-structure non-invasively. However, HARDI methods normally require many more samples than DTI. For example, Diffusion Spectrum Imaging (DSI)[1], estimates the diffusion Ensemble Average Propagator (EAP) from typically 515 diffusion-weighted images, taking nearly one hour. Therefore, one important area of research in HARDI is to improve estimation of quantities such the diffusion propagator, the orientation distribution function, and diffusion-weighted signals from ra educed number of diffusion-weighted measurements. A solution to this problem is compressed sensing (CS). Existing approaches to compressed sensing diffusion MRI (CS-DMRI) mainly focus on applying CS in the *q*-space of diffusion signal measurements**.** For example, it has been shown in [2] that it is possible to accurately estimate the diffusion signal, with low root-mean-square-error (RMSE), from about 100 diffusion-weighted measurements in multiple shells (multiple b-values). However, this line of work fails to harness information redundancy in the *k*-space. By sub-sampling in both *k*-space and *q*-space, the scanning time can be significantly reduced, while keeping good estimation quality. To our knowledge, [3,4] are the only works using joint k-q space CS schemes to estimate the fiber ODF from single-shell data. The utility of this joint CS approach in multi-shell acquisition has not been sufficiently studied. In the current work, we propose a general framework, called k-q compressed sensing diffusion MRI (kq-CS-DMRI), to estimate the diffusion signal and the diffusion propagator from data sub-sampled in both k-space and q-space.

Theory: **Naive CS-dMRI in k-q space**. In the last several years, CS has been widely used to recover MR images from data sub-sampled in the k-space [5]. Typically, each diffusion-weighted image is first independently reconstructed using k-space CS [5]. All estimated DW images are then used in for CS estimation of signal in the q-space [4]. This approach however ignores the correlation *between* DW images. Our approach, described below, unifies CS in both k-space and q-space.

Theory: Unified kqs-dMRI. Denote $\boldsymbol{Q}_v$ as the partial Fourier sample vector of v-th volume $\boldsymbol{E}_v$. Denote $\boldsymbol{E}_i$ as the vector of the diffusion weighted signals at voxel i. Assume diffusion signal vector $\boldsymbol{E}_i$ can be sparsely represented by a continuous basis set $\boldsymbol{B}$, and $\boldsymbol{c}_i$ is its representation coefficient vector. Then the estimation of the DW images via $\{\boldsymbol{c}_i\}$ can be reconstructed by their partial Fourier samples by minimizing:

$$\sum_{v=1}^{N_v}\left\{\left\|\boldsymbol{F}_p E_v - Q_v\right\|^2 + \lambda_1 TV\{E_v\} + \lambda_2\left|\Psi E_v\right|\right\} + \lambda_3\sum_{i=1}^{I}\left|\boldsymbol{c}_i\right|, \qquad \text{s.t.} \quad \boldsymbol{B}\boldsymbol{c}_i = \mathrm{Re}\{E_i\} \quad \forall i \in I$$

where the variables are $\{\boldsymbol{c}_i\}$ which represents the real part of $\{\boldsymbol{E}_i\}$. The difference between this formulation and the formulation in sparse MRI is that here we consider the relationship between different volumes $\boldsymbol{E}_v$. Thus we cannot naively perform sparse MRI separately for each volume. We use Alternating Direction Method of Multipliers (ADMM) [6] to solve it, which separates the optimization into three steps:

1. $c_i^{(k+1)} = \arg\min \lambda_3|c_i| + 0.5\rho\left\|Bc_i - E_i^{(k)} + U_i^{(k)}\right\|^2$; 2. $E_v = \arg\min\left\|\boldsymbol{F}_p E_v - Q_v\right\|^2 + \lambda_1 TV\{E_v\} + \lambda_2|\Psi E_v| + 0.5\rho\left\|\tilde{E}_v(\{c_i^{(k+1)}\}) - E_v + U_v^{(k)}\right\|^2$ 3. $U_i^{(k+1)} = U_i^{(k)} + Bc_i^{(k+1)} - E_i^{(k+1)}$

where the $\boldsymbol{U}$ is the Lagrangian multiplier which also takes into account the phase part of $\{\boldsymbol{E}_i\}$, k means k-th iteration step. The first sub-problem is LASSO which can be solved easily for each voxel. The second one is a modified version of sparse MRI, where the last term is the diffusion signal generated by $\{\boldsymbol{c}_i\}$ and we still use ADMM to solve it.

Theory: **Learned dictionary using SPF basis (DL-SPFI)**. In above formulation, we do not assume a specific form for the basis $\boldsymbol{B}$. We perform the state-of-the-art dictionary learning method to learn a dictionary $\boldsymbol{B}$ parameterized by the Spherical Polar Fourier basis. See [2] for the details. The learned dictionary in [2] has two advantages. (1) it satisfies the constrains that the normalized diffusion signal is 0 when b=0. (2) it uses adaptive scale to enhance the representation ability.

Synthetic experiments : In order to evaluate the proposed kqs-dMRI, we generate a slice of synthetic phantom with full size 20x20x1x514 using mixture of tensor model and DSI sampling scheme. Fourier transform is performed to obtain the ground truth of forier samples in k-space. Then 3 times reduction in k-space and 3 times reduction in q-space (totally 9 times) was done to obtain the subsampled raw data in k-space. First naive kq-CS-MRI was performed by first performning sparse MRI to estimate the full sampled data with size 20x20x1x170, then DL-SPFI [2] was performed to estimate the continuous signal and EAP. Second, we perform the proposed ADMM method to estimate the EAP. The ground truth of EAP at radius 15 *um* and the results from two methods can be seen in Fig.1. Compared to the naive way, the joint way obtains sharper EAP profiles, similar with the ground truth. Although we did not show the RMSE in the figure, the joint way has about 5% lower RMSE.

Real public data experiment: We downloaded one slice of a public real data from http://web.mit.edu/berkin/www/software.html. The data has been also used in [2]. Since there is no ground truth for this real data, especially for large b values where the signal-to-noise ratio is low. Thus we first performed DL-SPFI [2] on the given full sampled data, and used the reconstructed signal with size 96x96x1x514 as ground truth. Then using the same way as the synthetic data experiment, we generated the ground truth of data in k-space with size 96x96x1x514, and performed 3 times reduction in k-space and q-space respectively. The sub-sampled data was used in naïve k-q space estimation and the proposed joint framework. The RMSE compared to the synthetic ground truth can be seen in Fig.2. The joint framework solved by ADMM has generally lower RMSE.

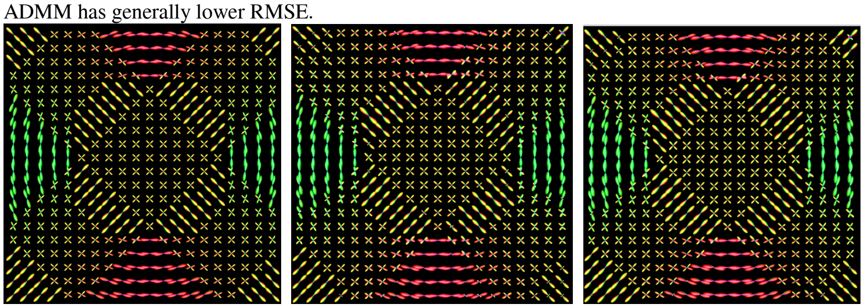

Fig.1 Ground truth at 15 *um* Naïve estimation kq-CS-DMRI

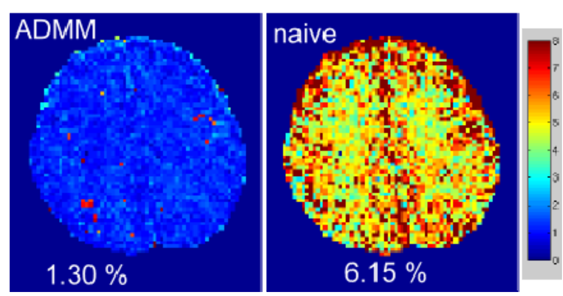

Fig.2 Real data

Conclusion: We propose a unified framework which combines both sparse MRI in k-space and compressed sensing in q-space. The unified cost function was solved using ADMM, where the two steps of compressed sensing estimation in two spaces are benefited by each other. To our knowledge, this is the first work on the joint k-q space compressed sensing diffusion MRI (kq-CS-DMRI) to estimate continuous diffusion signal and EAP from multiple shell data. Our experiments showed that compared to the fully sampled k-q space using DSI sampling scheme, it is possible to obtain similar estimation quality like DSI or SPFI by reducing 3 times samples in k-space and 3 times samples in q-space, totally 9 times reduction of scanning time. Compared to DSI, which normally needs one hour scanning time, our study showed that it is possible to obtain comparable estimation results from less than 10 minutes scanning time.

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