

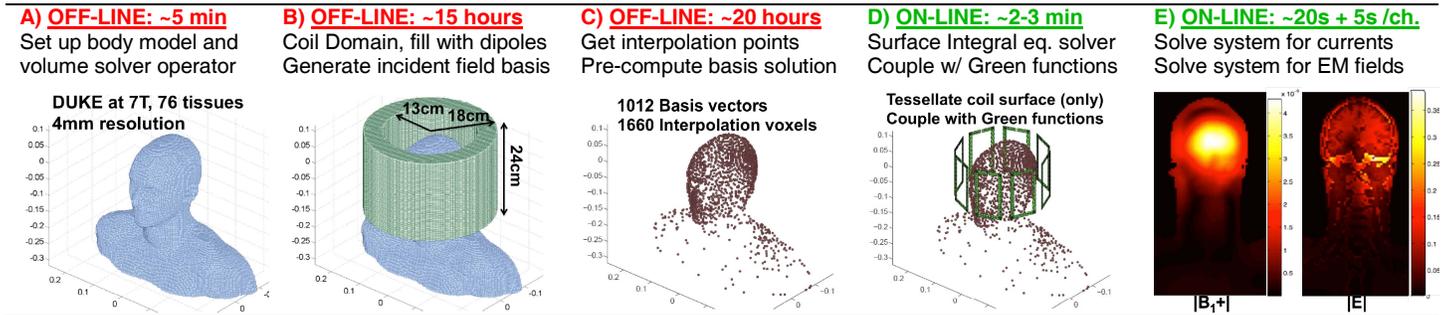
# Fast Electromagnetic Analysis of Transmit RF Coils based on Accelerated Integral Equation Methods

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**Target audience:** RF engineers and MR physicists. **Purpose:** In high-field scanners, the electromagnetic (EM) fields generated by the currents in transmit RF coils strongly depend on the sample. Therefore, for full wave EM analysis to be accurate enough to assess candidate coil designs, the simulated region must include both the coil design and a detailed body model [1,2]. For traditional finite-difference time-domain (FDTD) or finite-element based EM tools, simulating such a large region can take so long (an hour or more [2]) that coil design exploration is severely limited. Alternatively, we propose a methodology, based on a compressed body-model-specific Green function, that performs comprehensive **full-wave EM analysis of arbitrary MRI transmit coils, for a fixed realistic body model and frequency, in few minutes** -- fast enough to be used in automated coil optimization.

**Method:** Our approach relies on integral equation methods and a natural decomposition into two domains: 1) the coil array (including the shield), and 2) the realistic human body model. In order to accelerate the analysis of many coil designs for a fixed body model, we pre-compute a representation of this body model as a compressed Green function, in a slow "off-line" phase. We then use and reuse the representation in a fast "on-line" phase, to analyze as many candidate designs as desired. We assume that a given body model at a given frequency will be used to analyze a wide variety of coil designs, and that all candidate coil designs can be contained in a predefined "coil domain" (in this work a cylindrical shell just outside the body, valid for most of the head coil designs, although could be extended to more complex volumes outside the body).



The off-line phase (figures A) to C)) is done once for each frequency and body model, and involves the following steps. A) A voxel-based inhomogeneous realistic human body is given, e.g. DUKE [3] 4mm at 7T. B) The coil domain is defined as a volume external to the body, in which any coil design will be placed, e.g. a cylindrical shell as presented, valid for most traditional head coil designs. The coil domain is filled with current dipoles pointing in all three Cartesian directions. These dipoles are randomly excited and used to generate a basis for the incident electric field in the body, based on Randomized Singular Value Decomposition approaches [4]. This incident field basis can approximate any illuminating field generated by any current distribution for any coil design, as long as the design is contained in the defined coil domain. The accuracy of the approximation can be monitored and controlled when generating the vectors of the basis. C) The solution of the EM scattering problem is computed for the basis. A fast frequency domain Volume Integral Equation solver [5] can be applied to obtain the E and B fields in each body voxel due to an illuminating field, represented by each vector of the basis. A Discrete Empirical Interpolation Method [4] is then applied to obtain a reduced set of interpolation points that can be used to compute basis vector coefficients.

The result of the off-line phase is a set of interpolation voxels in the body plus a set of low-rank matrices, which form a compressed pre-computed body-model-specific Green function. This function (computed once), allows us to reconstruct the EM effect of the body for any coil design within the predefined coil domain. In the on-line phase, we will reuse these Green functions with any given coil design. Figures D) and E) represent the workflow for a given coil design under investigation. Fig. D), a Surface Integral Equation model, which only needs to tessellate the surface of the conductors, is generated for the given coil design, and coupled with the pre-computed Green functions, which mimic the effect of the body by a set of low-rank matrix products. Fig. E), we can solve this system to get **coil current distribution, and the EM fields in all the body model voxels within seconds**.

**Results and Discussion:** The approach is applied in a head and shoulder 4mm resolution DUKE model at 300MHz. Using the described cylindrical coil domain, the Green functions are pre-computed off-line in 35h, generating 1012 basis vectors and 1660 interpolation points for the body model (out of the original 205526 voxels), and a set of low-rank matrices, with total storage requirement of less than 10GB. We compare the on-line performance of the proposed approach (Pre-computed) and the solution by coupling Surface and Volume Integral Equations (Coupled IE). We test both approaches on two different coil arrays; one with 8 planar coils (shown in D), and one with 16 conformal coils. The table presents the number of unknowns and time required to assemble the coil models, the number of unknowns of the body model (all the body voxels for the Coupled IE, and the pre-computed interpolation points), the time required to solve the system to obtain the current distribution in the coils, and, once the currents are computed, the time to obtain the EM field map distribution for each coil channel. The error of the proposed approach is measured in maximum relative error for the coil currents, and the maximum of the RMSE for the electric and magnetic fields in the body, with respect to the Coupled IE. For the results presented, all approaches were implemented in MATLAB and run in a single server with two E5-2687W processors.

Coil Design	Method	Coil Model	Body model	Solve for the currents	Solve for EM fields
8 Planar Coils Array	Coupled IE	3322 unknowns, 110 sec.	205526 voxels	1668 sec. / port	112 sec. /ch.
	Pre-computed	3322 unknowns, 110 sec.	1660 interp.	15 sec. overall, < 0.03% Error	5 sec. /ch. < 0.38% RMSE
16 Conformal Coils Array	Coupled IE	4362 unknowns, 166 sec.	205526 voxels	2187 sec. / port	114 sec. /ch.
	Pre-computed	4362 unknowns, 166 sec.	1660 interp.	20 sec. overall, < 0.17% Error	5 sec. /ch., < 0.42% RMSE

The proposed approach speed is now limited by the coil model generation, with speed-ups near 100x w.r.t. the Coupled IE approach and negligible errors that can be controlled in the pre-computation stage. This approach also allows us to evaluate the effects of subject positioning within the coil array by reusing the coil model, and thus estimate the effect of changes in the body position in the scanner within seconds. It also allows pre-computation and storage of the Green functions for different body models, using them to analyze a given coil design performance in different body models.

**References:** [1] Guerin et al, MRM 2013. [2] Kozlov et al, JMR 2009. [3] Christ et al, PMB 2010. [4] Hochman et al, TAP (submitted). [5] Polimeridis et al, JCP (submitted). **Acknowledgements:** NIH support: 1R01EB0006847. Siemens-MIT CKI Alliance. Skoltech Initiative on computational mathematics. Swiss NSF PA00P2-142060.