

The best of both worlds: Improved rapid quantitative susceptibility mapping (QSM) by combining closed-form L2 regularization with SDI

Ferdinand Schweser¹, Andreas Deistung¹, Xiang Feng¹, Edsel Daniel Peres Gomez², and Jürgen R Reichenbach¹

¹Medical Physics Group, Institute of Diagnostic and Interventional Radiology I, Jena University Hospital - Friedrich Schiller University Jena, Jena, Germany, ²Medical Physics Group, Institute of Diagnostic and Interventional Radiology I, Jena University Hospital - Friedrich Schiller University Jena, Germany

TARGET AUDIENCE – Researchers interested in algorithmic aspects of rapid quantitative susceptibility mapping (QSM).

PURPOSE – Quantitative susceptibility mapping (QSM) is a recent post-processing technique for gradient-echo phase data¹. Involving several sophisticated processing steps QSM yields quantitative maps of tissue magnetic susceptibility, showing unprecedented anatomical contrast of deep brain regions¹. Additional to the unique anatomical depiction of deep brain regions¹, e.g. for stereotactic targeting, and its many clinical applications such as lesion characterization³, QSM promises to provide valuable information on tissue iron content⁴. While early efforts in algorithmic developments focused on improving the reconstruction quality, i.e. attenuating the so called “streaking artifacts” and limiting noise amplification^{5,6}, more recent efforts focus on accelerating the computation time of QSM to bring the technique into the clinics^{7,8}: Schweser et al.⁷ have recently presented that inverse filtering with extreme thresholding of the unit dipole response yields susceptibility maps within seconds without noise amplification and with a low level of streaking artifacts.⁷ The method, dubbed Superfast Dipole Inversion (SDI) does not rely on smoothing priors and, thus, provides images with a rather natural appearance.⁷ Bilgic et al.⁸ presented a closed-form solution for rapid L2-regularized susceptibility mapping with a gradient-based penalty term, referred to in the following as closed form L2 (CF-L2) algorithm. Due to the gradient penalty term the technique provides images with reduced noise level compared to SDI (see below). **In this contribution, we show, first, that combining CF-L2 with SDI can considerably reduce reconstruction artifacts of CF-L2 and, second, that CF-L2 produces sharper images if the three-point gradient is used for the regularization instead of the two-point gradient.**

THEORY – Both SDI and CF-L2 solve the inverse field-to-susceptibility problem based on the same mathematical principle, that is, inverse filtering. Inverse filtering means that the susceptibility distribution χ is obtained by applying a certain inverse filter to the field distribution ΔB (derived from the MR phase), which can be efficiently computed by point-wise multiplication ($/$) in the Fourier domain: $\chi \sim \text{iFT}\{ \text{FT}(\Delta B) / D \}$, where D is the Fourier-domain filter kernel. The SDI-kernel D_{SDI} is a completely thresholded version of the unit-dipole response D_{orig} (that describes the forward problem). The CF-L2 kernel $D_{\text{CF-L2}}$ is a more complex kernel that involves two-point gradient kernels as well as the (unmodified) unit-dipole response, D_{orig} . We propose to combine CF-L2 and SDI by using D_{SDI} instead of D_{orig} when constructing $D_{\text{CF-L2}}$. The resulting method is dubbed SDI-CF-L2 in the following. In addition, we propose to evaluate the gradients in $D_{\text{CF-L2}}$ over three spatial points (three-point gradient) instead of 2-points.

METHODS – *Data Acquisition and pre-processing*: Double-echo gradient echo data were acquired from six healthy volunteers (male; 26-28 year old) on a 3T whole-body MRI scanner (Tim Trio, Siemens Medical Solutions, Erlangen, Germany) with the ToF-SWI-sequence⁹. The institutional review board approved the study and informed written consent was obtained from all subjects. The following sequence parameters were used: $TE_1/TE_2 = 12\text{ms}/40\text{ms}$, $TR=46$ ms, $FA=20^\circ$, voxel size= $470\mu\text{m} \times 470\mu\text{m} \times 940\mu\text{m}$. Multi-channel images were combined using uniform sensitivity reconstruction¹⁰. Phase pre-processing (unwrapping and background correction) was performed according to the SDI framework⁷. *QSM reconstruction*: The following different QSM-algorithms were compared: A) SDI, B) CF-L2 with 2-point gradients (the original CF-L2), C) SDI-CF-L2 with 2-point gradients, D) SDI-CF-L2 with 3-point gradients. With all algorithms (A-D) susceptibility maps were calculated from all subjects. For CF-L2 the optimal regularization parameter of 0.015 (from L-curve optimization in Ref. 8) was used.

RESULTS – **Figure 1** compares susceptibility maps obtained from SDI and CF-L2 with maps from the *proposed* SDI-CF-L2. The CF-L2 susceptibility map has a considerable lower noise level compared to the SDI map and an overall smoother appearance due to the involved gradient regularization. However, the CF-L2 suffers from slowly varying inhomogeneities (red arrows in Fig. 1). These inhomogeneities are successfully removed in the susceptibility map resulting from the proposed SDI-CF-L2, which is also illustrated by the difference image of the CF-L2 and SDI-CF-L2 maps in Fig. 1 (right-most image). **Figure 2** compares SDI (left) with SDI-CF-L2 using 2-point gradients (middle) and 3-point gradients (right). Both SDI-CF-L2 algorithms produce susceptibility maps with considerably reduced noise level compared to SDI. However, the 3-point algorithm produced maps with a slightly crispier visual appearance. Computation time was on the order of seconds with all algorithms. Similar results (as shown in Figs. 1 and 2) were obtained in all other volunteers.

DISCUSSION – Our results demonstrate that CF-L2 yields susceptibility maps with decreased noise level compared to SDI, however, with increased inhomogeneities. When analyzing these inhomogeneities in more detail we found that they propagated from the background-field corrected phase images, e.g., due to erroneous masking. This indicates that CF-L2 is actually more sensitive to errors in the input phase data than SDI, despite the involved gradient regularization. The combination of both techniques yields susceptibility maps with a lower noise level than SDI and without the inhomogeneity artifacts. The visual smoothing of CF-L2 was slightly reduced by changing the gradient type from 2-point to 3-point evaluation, which can be explained by the fact that the 3-point gradient is less sensitive to tissue interfaces. Future studies will compare the techniques against gold standard susceptibility maps such as numerical brain models or susceptibility maps calculated with more sophisticated (and computationally more expensive) algorithms⁶.

CONCLUSION – Combining CF-L2 with SDI yields substantially improved quantitative susceptibility maps without increasing the computation time.

REFERENCES – [1] Schweser F et al., 2011. *NeuroImage*. 54(4):2789-807. [2] Schweser F et al., 2010. *Med Phys*. 37(10):5165-78. [4] Langkammer C et al., 2012. *NeuroImage*. 62(3):1593-9. [5] Liu J et al., 2012. *NeuroImage*. 59(3):2560-8. [6] Schweser F et al., 2012. *NeuroImage*. 62(3):2083-100. [7] Schweser F et al., 2013. *Magn Reson Med*. 69(6):1581-93. [8] Bilgic B et al. *J Magn Reson Imag* (epub). [9] Deistung A et al., 2009. *J Magn Reson Imag*. 29(6):1478–84. [10] Ros C et al., 2009. *IFMBE Proc* 22:803-6.

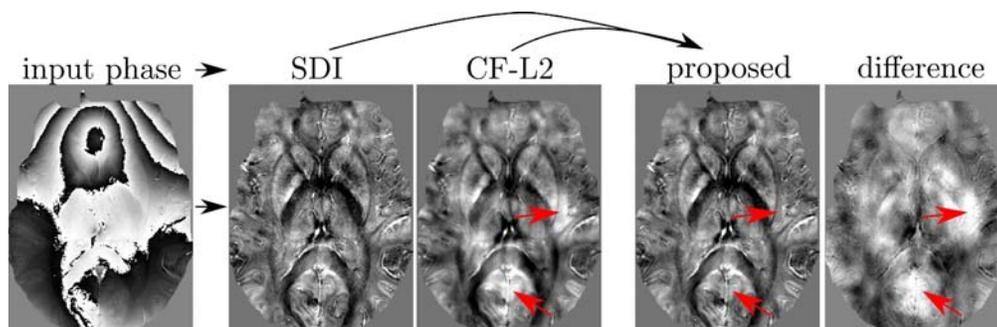


FIGURE 1. Comparison of SDI, CF-L2 and the proposed combination of SDI and CF-L2 (contrast is -0.1 to 0.2 ppm). The difference between CF-L2 and the proposed combination is shown in the right-most figure (contrast is -0.05 to 0.05 ppm). The combination substantially attenuated inhomogeneities visible in CF-L2 (red arrows).

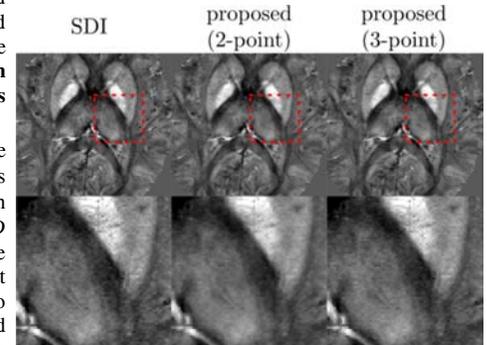


FIGURE 2. Comparison of SDI (left) and CF-L2 with 2-point gradients (middle) and with 3-point gradients. The bottom row shows enlarged sections marked in the top row by red squares.