# Modulated closed form solution for quantitative susceptibility mapping

Khabipova Diana[1], Rolf Gruetter[1,2], and Marques P. José[3]

[1]*LIFMET, EPFL, Lausanne, Vaud, Switzerland,* [2]*Radiology, University of Geneva, Geneva, Geneva, Switzerland,* [3]*Radiology, University Lausanne, Lausanne, Vaud, Switzerland*

**INTRODUCTION** Quantitative susceptibility, $\chi$, mapping (QSM) has shown the potential to deliver estimates of iron deposition in deep grey matter structures. Because it is an ill-posed problem, many regularization methods have been proposed [1-4] yet, most methods remain time consuming, rely on careful regularization selection and their results are inferior to those obtained using multiple orientation acquisitions (COSMOS)[5]. Recently, a closed-form solution for the QSM including an l2-regularization of the gradient of the reconstructed susceptibility map was proposed [6] which allows a fast reconstruction. In this abstract we demonstrate the advantage of using a k-space position dependent modulation of the gradient in order to avoid regularizing the QSM solution in regions where the QSM is not ill-posed.

**THEORY:** The closed-form (CF) solution described in [6] relies on the description of the gradient along a direction *i* as $\vartheta_i = F^H E_i F$, where F is Fourier Transform, $E_i$ is given by $E_i = 1 - e^{-2\pi i k_i / N_i}$, and $k_i$ is the k-space coordinate along *i* direction. This method is extremely fast but, when compared to standard iterative methods[2-5], the application of the gradient regularization in the whole image (and k-space), gives rise to smoother $\chi$ maps. To overcome these limitation, a weighting in the k-space of the regularization term was introduced to ensure that the regularization is only applied on the ill-conditioned k-space points. The final expression of the modulated closed-form (MCF) solution can be written as $\chi(k) = \frac{D(k)}{D(k)^2 + \lambda^2 \Lambda(k)^2 \sum_i E_i^2} \delta B(k)$, [Eq.1] where D(k) is the k-space representation of the dipole kernel, $\lambda$ is a regularization parameter, and $\Lambda$(k) is a weighting matrix defined as $\begin{cases} \cos\left(\frac{\pi D(k)}{n_{th}}\right), & D(k) < n_{th} \\ 0, & D(k) \geq n_{th} \end{cases}$ ,[Eq.2]

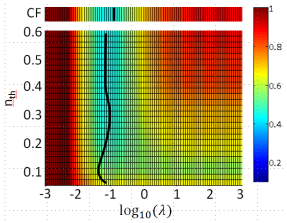


**Fig. 1** shows error of QSM maps in respect to Ground Truth (zero being lowest reconstruction error) as a function of the regularization parameters λ (see Eq. 1) and nth (see Eq. 2), black line denotes the best solution based on the L-curve heuristic

**METHODS:** Numerical simulations: As in [7], the performance of the QSM method was evaluated as a function of the regularization and threshold parameter using a numerical phantom of known susceptibility and its fieldmap corrupted with noise.

Experimental Protocol: Three healthy volunteers were scanned on a 7T scanner (Siemens) according to a protocol accepted by the local ethics committee. Whole brain 3D GRE data was acquired: TR/$TE_1$-$TE_5$=42/4.97/37.77ms, BW=260 Hz/Px, α=10°, isotropic res= 0.66mm, iPAT=2x2, $T_{acq}$=11 mins. The scan was repeated 4 times with the subject's head oriented along different orientations. The relative head positions was computed by co-registering the different head positions using FSL-FLIRT. Data from the different 32 coils was combined using a SVD factorization of the channel vs. echo time matrix. The phase images were unwrapped, a field map was computed and the background field was removed using an Harmonic Field Pre-Filtering [8]. $\chi$ maps were calculated with a large range of parameters λ and $n_{th}$. The optimum parameters were determined by using the maximum second derivative of the L-curve.

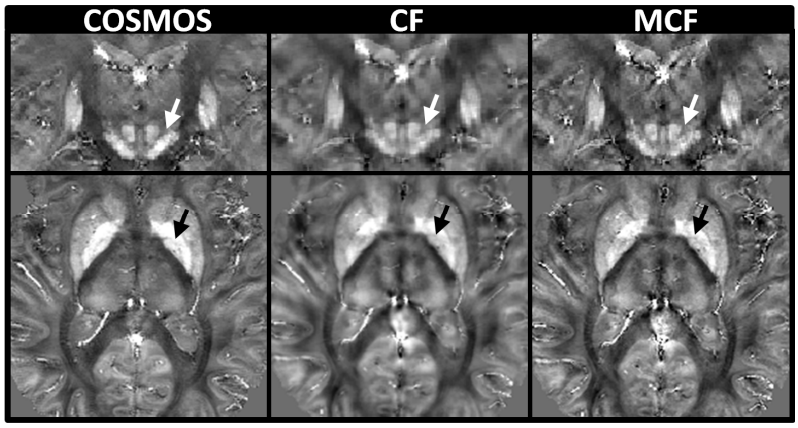


**Fig. 2** Representative coronal (top row) and transverse (bottom row) slices of $\chi$ maps obtained using the COSMOS (1st column), the closed form solution [6] (2nd column) and the proposed MCF method (3rd column). The regularization parameters were chosen using the L curve heuristic (CF:λ=1; MCF: λ=1.6, $n_{th}$=0.2). Using COSMOS and MCF method the SN and RN borders (white arrows) and lamina pallidi (black arrows) can be easily distinguished, but not with CF method.

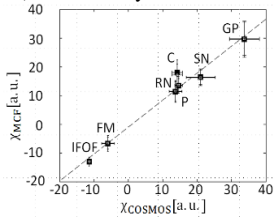


**Fig. 3** shows the correlation between COSMOS and the susceptibility maps obtained with the CF solution (Rho = 0.94, $r^2$ = 0.97) on the regions of interest, GP -globus pallidus, SN - susbtantia nigra, C - caudate, RN - red nucleus, P - putamen, FM - forceps major, IFOF - inferior fronto-occipital fasciculus

**RESULTS:** Fig. 1 shows that the reconstruction error in simulated data remains constant throughout different thresholds for the optimal regularization parameter λ but, choosing a modulation mask (nth=0.2) makes the MCF method more independent from the regularization parameter, see Fig. 1. Furthermore, the L-curve based estimation of the optimum parameters is a good estimator of the optimum λ in the absence of ground truth.The best susceptibility maps evaluated using the L-curve heuristic are shown in Fig. 2 for the CF and MCF methods as well as those obtained using COSMOS method[5]. The correlation between COSMOS and MCF on anatomical regions for the three subjects demonstrates the quality of the methods (Rho = 0.94, $r^2$ = 0.97) which is not the case for most direct inversion methodologies [9].

**DISCUSSION/CONCLUSION:** The results show that our new QSM methodology is effective at calculating susceptibility values in deep gray matter regions without showing signs of over-smoothing the computed QSM maps. A choice of a $n_{th}$ =0.2 makes the reconstruction largely independent from the regularization parameters, with the reconstruction maps being comparable for λ values an order of magnitude greater than optimum. This feature makes it ideal in the context of unsupervised usage when applying QSM to a large groups of subjects .

**REFERENCES:** [1] K. Shmueli et. al., MRM, vol. 62, 2009.[2] L. De Rochefort et. al., MRM, vol. 63, 2010. [3] B. Bilgic et. al., NeuroImage, vol. 59, 2012.[4] T. Liu et. al., MRM, vol. 66, 2011.[5] T. Liu et.al.,MRM, vol.61, 2009.[6] B.Bilgic et al., JMRI, 2013.[7] D.Khabipova et. al., Proc.ISMRM, 2493, 2013.[8] L. de Rochefort et al., Proc.ISMRM, 170, 2013. [9] S. Wharton et. al., NeuroImage, vol. 53, 2010

**ACKNOWLEDGEMENT** This work was supported by CIBM of the UNIL,UNIGE, HUG, CHUV, EPFL and the Leenaards and Jeantet Foundation.