Fast Reconstruction for Regularized Quantitative Susceptibility Mapping

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TARGET AUDIENCE: Physicians and scientists interested in Quantitative Susceptibility Mapping (QSM) and phase imaging.

PURPOSE: QSM estimates the underlying magnetic susceptibility of tissues that give rise to changes in the magnetic field, and has applications in tissue iron quantification [1] and vessel oxygenation estimation [2]. ℓ_1- and ℓ_2-regularization have been proposed [3,4] to help solve the ill-conditioned dipole inversion in QSM. A high-resolution whole brain QSM reconstruction can take up to 20 min on a workstation, which poses a limit on QSM usability in clinical and research settings. Recently, the use of Split-Bregman (SB) formulation with ℓ_1-regularization [5] and an efficient closed-form solution to the ℓ_2-regularized problem [6] have been proposed to significantly decrease the computation cost of these problems. Herein, we introduce an improved SB ℓ_1-regularized dipole inversion algorithm that offers 20× faster reconstruction relative to the standard nonlinear conjugate gradient (NCG) solver. This fast reconstruction renders estimation of regularization parameters with the L-curve heuristic feasible. Additionally, we extend SB ℓ_1-regularization to admit magnitude-weighting that prevents smoothing across edges identified on the magnitude signal, and solve this more complicated problem 5× faster than the NCG approach. Further, we extend the previously proposed closed-form ℓ_2-based inversion [6] to admit magnitude-weighting, and demonstrate 15× acceleration relative to NCG by employing a preconditioner that leads to faster convergence. Utility of the proposed methods is demonstrated in high-resolution (0.6 mm isotropic) 3D GRE data at 3T, as well as multi-echo Simultaneous Multi-Slice (SMS) EPI time-series at 7T, wherein processing of such large datasets would otherwise be prohibitive with conventional NCG.

METHODS: Tissue susceptibility $\boldsymbol{\chi}$ relates to the measured field map $\boldsymbol{\phi}$ via $\mathbf{DF}\boldsymbol{\chi} = \mathbf{F}\boldsymbol{\phi}$, where $\mathbf{F}$ is Fourier transform operator and $\mathbf{D}$ is the dipole kernel. **$\boldsymbol{\ell_2}$-regularized QSM:** aims to solve $min\|\mathbf{F}^{-1}\mathbf{DF}\boldsymbol{\chi} - \boldsymbol{\phi}\|_2^2 + \beta\|\mathbf{WG}\boldsymbol{\chi}\|_2^2$ where $\mathbf{G}$ is the gradient operator and $\mathbf{W}$ is a binary mask derived from the gradient of the magnitude image. The optimizer is given by the solution of $(\mathbf{F}^{-1}\mathbf{D}^2\mathbf{F} + \beta\mathbf{G}^T\mathbf{W}^2\mathbf{G})\boldsymbol{\chi} = \mathbf{F}^{-1}\mathbf{DF}\boldsymbol{\phi}$. Without magnitude weighting ($\mathbf{W} = \mathbf{I}$), it can be computed in closed-form [3] as $\boldsymbol{\chi} = \mathbf{F}^{-1}(\mathbf{D}^2 + \beta\mathbf{E}^2)^{-1}\mathbf{DF}\boldsymbol{\phi}$, by expressing the gradient as $\mathbf{G} = \mathbf{F}^{-1}\mathbf{EF}$ where $\mathbf{E}$ is a diagonal matrix. Since the inversion involves only diagonal matrices, it requires only two FFTs. With magnitude weighting, the linear system is no longer diagonal. We propose to use the closed-form solution as preconditioner and iteratively solve the modified system $(\mathbf{D}^2 + \beta\mathbf{E}^2)^{-1} \cdot \{(\mathbf{D}^2 + \beta\mathbf{A}) \cdot \mathbf{F}\boldsymbol{\chi} - \mathbf{DF}\boldsymbol{\phi}\} = \mathbf{0}$ where $\mathbf{A} = \mathbf{E}^H\mathbf{FW}^2\mathbf{F}^{-1}\mathbf{E}$. As the weight matrix $\mathbf{W}$ contains only the strongest edges, it is equal to identity $\mathbf{I}$ except for ~5% of its entries [3]. This makes the approximation $(\mathbf{D}^2 + \beta\mathbf{E}^2)^{-1} \approx (\mathbf{D}^2 + \beta\mathbf{A})^{-1}$ valid, and renders the preconditioner useful. **$\boldsymbol{\ell_1}$-regularized QSM:** we extend the SB formulation [7] to QSM by solving $min\, 1/2\|\mathbf{F}^{-1}\mathbf{DF}\boldsymbol{\chi} - \boldsymbol{\phi}\|_2^2 + \lambda\|\boldsymbol{y}\|_1 + \mu/2\|\mathbf{y} - \mathbf{WG}\boldsymbol{\chi}\|_2^2$. At iteration t, $\boldsymbol{\chi}$ and $\mathbf{y}$ are updated due to (*i*) $(\mathbf{D}^2 + \mu\mathbf{A})\mathbf{F}\boldsymbol{\chi}_{t+1} = \mathbf{DF}\boldsymbol{\phi} + \mu\mathbf{E}^H\mathbf{FW}^T\boldsymbol{y}_t$ and (*ii*) $\boldsymbol{y}_{t+1} = max(|\mathbf{WG}\boldsymbol{\chi}_{t+1}| - \lambda/\mu, 0) \cdot sign(\mathbf{WG}\boldsymbol{\chi}_{t+1})$. Without magnitude weighting, (*i*) can be rapidly solved in closed-form, while (*ii*) is a simple soft-thresholding step. With the inclusion of **W,** the preconditioner $(\mathbf{D}^2 + \mu\mathbf{E}^2)$ is employed for fast iterative solution via linear conjugate gradient. Using the susceptibility estimate from the previous iteration $\boldsymbol{\chi}_t$ as initial guess further improves convergence. **Data Acquisition:** 3D GRE at 0.6 mm iso res was acquired on a volunteer at 3T (TR/TE=26/8.1ms, $R_{inplane}$=2, T_{acq}=16min), and a multi-echo SMS EPI dataset at 2 mm iso res was collected on a healthy volunteer at 7T (TR/TE_1/TE_2/TE_3/TE_4=2040/15/35/54/74ms, $R_{inplane}$×MB=3x3). Phase data were processed with Laplacian unwrapping [8] and Sharp filtering [9]. β, μ and λ values were chosen using L-curve.

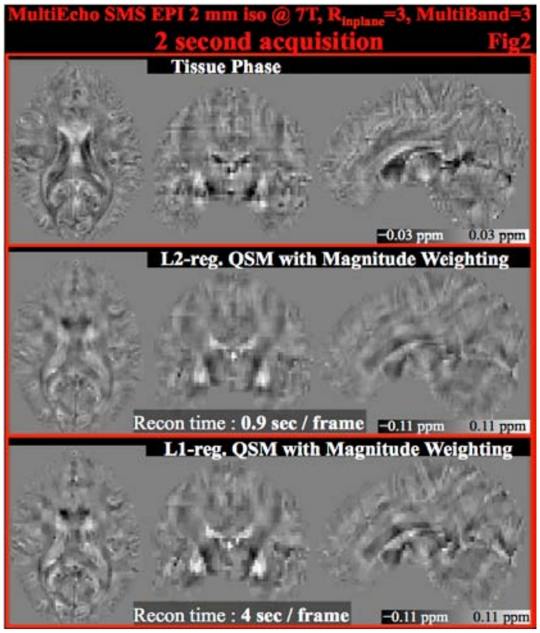

Fig2

RESULTS: **3D GRE:** Reconstruction with closed-form ℓ_2-regularization [6] took 0.9s, while the proposed ℓ_2-based inversion with magnitude weighting was completed in 88s (Fig1, top). Proposed ℓ_1-regularized QSM was finished in 60s and 275s without and with magnitude weighting (Fig1, bottom). Conventional NCG requires **1350s** to reach the same convergence criterion of less than 1% change in the $\boldsymbol{\chi}$ update (not shown). **SMS EPI:** ℓ_2- and ℓ_1-based dipole inversion with magnitude weighting are completed in 0.9s and 4s per frame in the time-series (Fig2 shows results for TE1 data).

DISCUSSION: The proposed dipole inversion algorithms dramatically reduce the processing time of ℓ_1- and ℓ_2-regularized QSM, while admitting prior information derived from the magnitude signal for edge-aware regularization. Such fast processing is made possible by efficient use of the closed-form ℓ_2-based solver as preconditioner and the variable splitting method that decomposes ℓ_1-penalty into a least-squares problem followed by a soft-thresholding step. While yielding up to 20× speed-up relative to conventional optimization methods, the proposed algorithms are further combined with fast phase unwrapping and background removal techniques to yield a rapid pipeline that might facilitate clinical application of QSM.

REFERENCES: [1] Langkammer C. et al. NIMG'12; [2] Fan A.P. et al. MRM'12; [3] Liu T. et al. MRM'10; [4] Liu J. et al. NIMG'12; [5] Chen Z. et al. Comp Assist Tomog'12; [6] Bilgic B. ISMRM'13; [7] Goldstein T. et al. SIAM'09; [8] Li W et al. NIMG'11; [9] Schweser F. et al. NIMG'11.

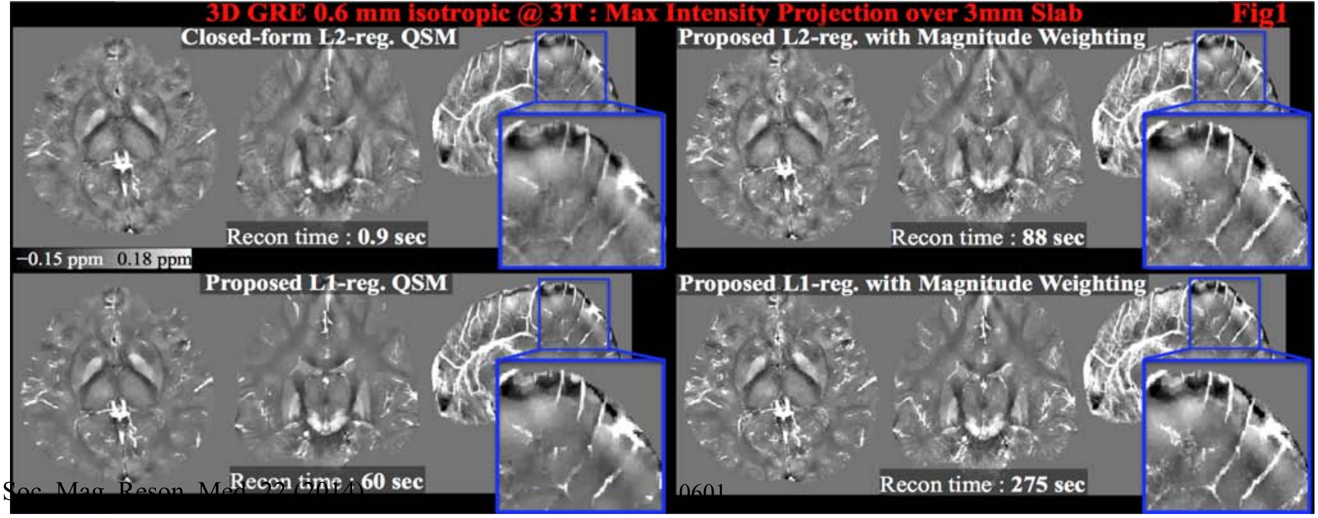

Fig1

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