Background Field Removal by Solving the Laplacian Boundary Value Problem

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Target Audience: Researchers interested in removing background magnetic field, such as for quantitative susceptibility mapping (QSM) and susceptibility weighted imaging.

Background: In QSM, the magnetic susceptibility is mapped out by solving the field-to-source inverse problem using the phase data. Since only MRI data in tissue region are available, the study of tissue property requires the elimination of the magnetic field caused by sources outside the tissue region [1-5]. This background removal directly affects the calculated susceptibility values [4], and is also a required pre-processing procedure for susceptibility weighted imaging [5].

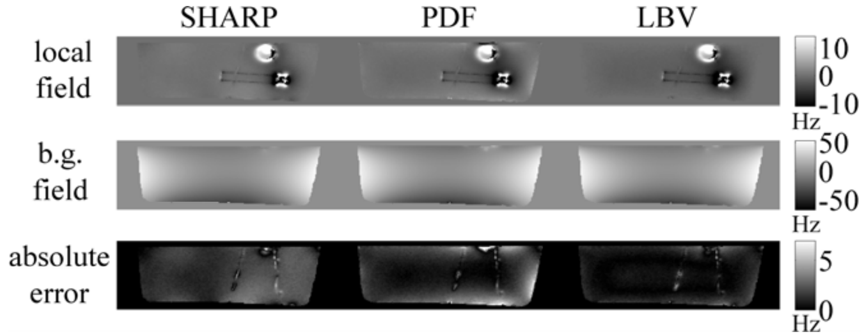

Figure 1. Local magnetic field of the phantom data.

Theory: Denote by f_T the measured total field (along and scaled to B_0) estimated from MRI data. Then Maxwell's Equations lead to $\nabla^2 f_T = (\nabla^2/3 - \partial_z^2)\chi$, where $\chi \ll 1$ is tissue susceptibility [6]. Inside a region of interest (ROI), $f_T = f_L + f_B$ with f_L the field caused by local tissue and f_B the background field caused by susceptibility sources outside ROI, ie, $\nabla^2 f_B = 0$ for points in the ROI. Existing methods such as PDF [2, 4] and SHARP [3] can be viewed as solvers for this partial differential equation (PDE). Here we remove the background field by directly solving the Laplace equation with specified boundary values (LBV). Because the boundary values are not easily available and the local field is typically one order of magnitude smaller than the background field, we make the approximation that $f_B = f_T$ on the boundary of the ROI. Similarly, the local tissue field f_L can be directly solved from a boundary value problem of Poisson's equation. Laplace's and Poisson's equations are well studied elliptic PDEs and their boundary value problems can be solved successfully by various schemes including finite difference, spectral, and finite element methods [7]. In this work, we use the full multigrid method.

Methods: A cylindrical water phantom with two vials containing Gadolinium solutions was scanned on GE 1.5T scanner with matrix size 256x256x80, voxel size 0.9375x0.9375x1.2 mm, field of view (FOV) 240 mm, flip angle 15°, bandwidth ±62.5 kHz, 10 echoes, TE1/dTE/TR=3.1/3.1/52.2 ms. Brain data of healthy subjects were acquired using the gradient echo pulse sequence on GE 3T scanner with the following parameters: matrix size 256x256x116, voxel size 0.9375x0.9375x1.2 mm, FOV= 240 mm, flip angle 20°, bandwidth 62.5 kHz, 7 echoes, TE1/dTE/TR = 2.8/5/37.5 ms. For the SHARP method, the k-space kernel threshold 0.01 was implemented as the 3D discrete Laplacian operator to retain data near the ROI boundary. The PDF, SHARP and the proposed LBV methods were implemented in C, C++ and C++ respectively.

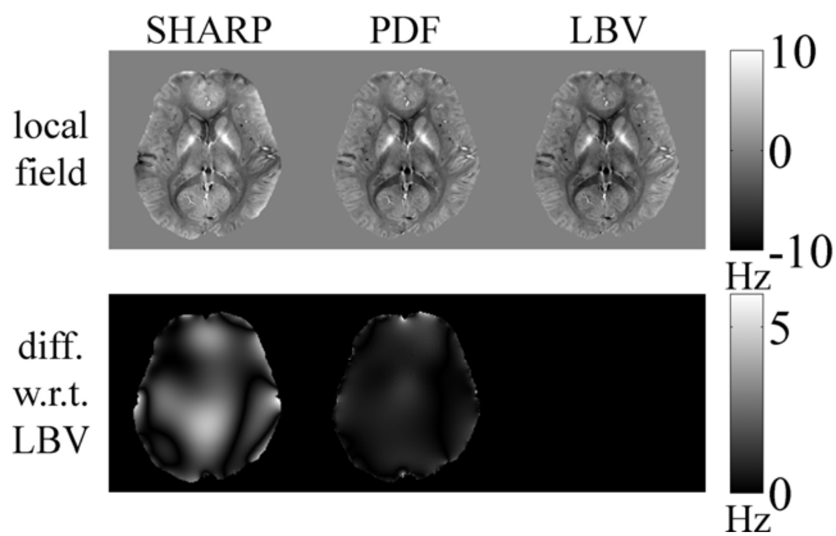

Figure 2. Local magnetic field of the brain data.

Results: In Figure 1 and 2, we show the local tissue field calculated from three different background removal methods. In the phantom study, the error of SHARP appears to have long wavelength variations. PDF and LBV have the largest error near the ROI boundaries. In the brain study, calculated local fields show overall agreement on tissue structures while differences are mostly evident on the ROI boundaries. The computation times of the brain data for the three methods are 5s (SHARP), 14s (PDF) and 6s (LBV).

Conclusion: We propose a new method to remove the background field by solving the PDE of the background magnetic field assuming simple boundary conditions. The proposed Laplacian boundary value (LBV) method for background field removal retains data near the boundary and is computationally efficient. Tests on an experimental phantom and in *in vivo* data sets showed that LBV was more effective than the SHARP and PDF methods.

References: [1] Marques and Bowtell 2005, Concepts in MR Part B (25) 65; [2] Liu et.al. 2011, NMR in biomedicine (24) 1129; [3] Schweser et.al. 2011, NeuroImage (54) 2789; [4] Wharton et.al. 2010, MRM (1304) 1292; [5] Haacke 2004 MRM (52) 612; [6] de Rochefort, et al, 2008, Med Phys .(35) 5328. [7] Press et.al. 2007, Numerical Recipes 3rd edition.