Power balance considerations for RF transmit coil arrays

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Target Audience: Researchers interested in EM simulation, safety evaluation and optimization of parallel transmission coil arrays.

Purpose: In this work we aim to extend the quadratic form power correlation matrix (PCM) formalism [1] used for estimation of deposited power in lossy materials by a transmit coil array. A distinct PCM is derived for each term of the power balance – forward, reflected, absorbed, radiated, and lumped element loss power. The goal is to enable a straight-forward calculation of losses for arbitrary excitations, transmit array worst-case loss analysis; and also to verify EM simulation integrity, which is imperative especially for complex workflows that are prone to mistakes.

Theory: The approach is based on the calculation of absorbed power P via quadratic forms $P = v^H \mathbf{Q} v$ for each term in the balance. In an N-channel array, $\mathbf{Q}$ denotes an $N \times N$ PCM, v the applied coil voltage column vector and H the hermitian transpose. Knowledge of the coil array scattering matrix $\mathbf{S}$, 3D $\mathbf{E}$- and $\mathbf{H}$ field distributions and lumped element (capacitor, inductor, resistor…) currents and voltages is assumed. All values are normalized to a unit excitation of 0.01W forward power, i.e. 1V amplitude applied voltage at a source impedance $Z_0 = 50\ \Omega$. The forward PCM, $\mathbf{Q}_F = (2Z_0)^{-1}\mathbf{1}$, used to calculate the power incident into the coil, is a scaled identity matrix. Any power loss due to coupling or imperfect matching is computed using the coupling loss matrix derived from the scattering matrix as $\mathbf{Q}_C = (2Z_0)^{-1}(\mathbf{S}^H\mathbf{S})$. For an N channel coil with M discrete lumped elements, the element voltages and currents can be arranged in $N \times M$ matrices $\mathbf{U}$ and $\mathbf{J}$, in which the Nth row contains the normalized voltages or currents of each lumped component for the excitation of coil element N. The resulting power loss is given as $P = 1/2\,\mathrm{Re}(v^H \mathbf{U}\mathbf{J}^H v)$, and resolving the real part yields $\mathbf{Q}_L = 1/4\ (\mathbf{U}\mathbf{J}^H + \mathbf{U}^H\mathbf{J})$ as the lumped element loss matrix. Power dissipated inside lossy materials is derived from the originally proposed PCM $\mathbf{Q}_D$, with its entries defined as $q_{ij} = 1/2 \int \sigma \mathbf{E}_i^* \mathbf{E}_j \mathrm{dV}$, with the conductivity σ and $\mathbf{E}_i$ denoting the electric field produced by a unit excitation of coil element i. By restricting the integration volume to certain materials (body, metal, substrate…), power correlation matrices can be obtained for each material of interest. A similar approach is taken for calculating the radiated power by integration of the pointing flux through a box enclosing the problem space. The radiated power of multiple interfering fields can be obtained via $P = 1/2\ \mathrm{Re}(v^H \widetilde{\mathbf{Q}}_R v)$, with the elements of $\widetilde{\mathbf{Q}}_R$ defined as $q_{ij} = \oint \mathbf{E}_j \times \mathbf{H}_i^* \mathrm{d}\vec{\mathrm{A}}$. Again resolving the real part yields $\mathbf{Q}_R = 1/4\,(\widetilde{\mathbf{Q}}_R + \widetilde{\mathbf{Q}}_R^H)$ for the radiated PCM. The power balance can now be written as $\mathbf{Q}_F = \mathbf{Q}_C + \mathbf{Q}_L + \mathbf{Q}_D + \mathbf{Q}_R$. Any residual imbalance of this equation can be attributed to factors such as non-convergence of FDTD simulations, insufficient sampling frequency, loss of precision due to simulation methodology or simply user errors. The by magnitude largest (smallest) Eigenvalue of the residual imbalance matrix represents the maximum (minimum) power balance error.

Methods: The presented theory was applied to an 8-channel array at 297.2 MHz (Fig. 1). Individual elements measuring 9x22 cm, constructed of 5mm wide copper strips, were arranged conformally on a cylindrical acrylic former (27cm outer diameter, thickness 1 cm) with a copper RF shield of 31 cm diameter. Nearest-neighbor decoupling (≈ -15 dB) was achieved using counterwound inductors [2]. The magnet bore was modeled as a 2.45 m long, 89.5 cm diameter steel cylinder. A spherical phantom, comparable to the human brain in dimensions as well as in dielectric properties $(d = 18\ cm, \epsilon = 50.6, \sigma = 0.66\ S/m)$, was used as the coil load. Coil simulation was done with XFdtd (Remcom, State College, PA, USA) being used for 3D EM simulations and ADS (Agilent, Santa Clara, USA) for tuning, matching and decoupling [3]. Particular care was taken to ensure accurate loss modeling. Metal losses were approximated by enabling a good conductor approximation in XFdtd [4]; and capacitors assigned an ESR of appropriate ATC 100E series models (American Technical Ceramics Inc., Huntington Station, NY, USA). Air core solenoid losses were estimated using wcalc (http://wcalc.sourceforge.net/) [5], and solder joint resistances were extrapolated from literature data [6] based on a √f frequency dependence. The loss tangent of the acrylic former was provided by XFdtd. Single channel fields and all power correlation matrices were calculated using MATLAB (The MathWorks, Natick, MA, USA). Finally, power imbalance, worst-case losses and the complete power balance for five different excitation modes were analyzed to demonstrate the method.

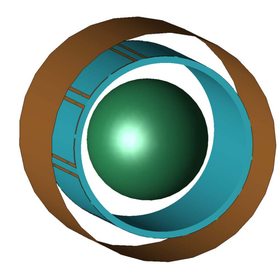

Fig. 1: Setup of the simulated coil array showing coil elements, acrylic former, phantom load and RF shield.

Results: The maximum and minimum power imbalances for the investigated array were 0.018% and 0.002%, respectively. The worst- and best-case losses for each power balance term are detailed in Table 1. Figure 2 shows the complete power balance for the four clockwise rotating excitations (“Birdcage”-Modes) and the “Maxwell”-Mode (equal phase for all coil elements).

Discussion: The maximum power imbalance is negligible and thus in excellent agreement with theory. The small residual error can be attributed to multiple factors. Determination of radiated power in FDTD simulations is prone to minor errors due to the use of a non-uniform mesh combined with the need to co-locate $\mathbf{E}$- and $\mathbf{H}$ fields for Poynting vector estimation via interpolation [7]. Furthermore, the complex simulation workflow and interplay of various calculation and post-processing routines can lead to additional errors due to limited precision of each computation and the exchanged data, such as S-Parameters. As care was taken to reach simulation convergence down to the numerical noise level (<-95 dB) and obey the sampling theorem w.r.t. the input pulse, these factors are not expected to contribute to the error. The reported imbalance can be regarded as a lower-limit error for simulations of comparable complexity. Power balance calculations for different excitation modes show a strong variation of the individual terms for different excitation patterns. The radiated power component is negligible in all investigated excitations but the common CP1+, for which it is similar to previously published results of comparable geometries [8]. Minimum and maximum absorbed powers are significantly different, and can vary by orders of magnitude for the radiated power component. Conductor metal losses still have an uncertainty attached, especially concerning the correct approximation of lateral skin effect contributions [9] in copper strip conductors. It may be advantageous to instead model coil conductor losses as lumped resistors derived from analytical considerations for the given conductor geometry and coil size, which has been shown to provide accurate estimates for intrinsic coil losses [6].

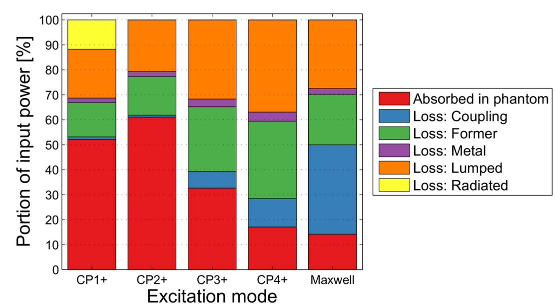

Fig. 2: Power balance for five different excitation modes of the array. The power distribution between different loss terms varies significantly between excitation modes and, as is to be expected, adds up to 100% within a negligible margin of error.

Conclusion: The presented theoretical framework has been shown to reliably provide access to the power balance for arbitrary excitations using simple matrix calculations. Worst case power imbalance estimates allow a straight-forward plausibility check for EM simulations of arbitrary complexity. Losses have been shown to vary significantly between different excitations, allowing a deeper insight into loss mechanisms than considering only a single mode.

	Phantom	Coupling	Former	Metal	Lumped	Radiated
max [%]	61.7	35.9	31.1	3.7	36.9	11.8
min [%]	14.2	0.6	13.8	1.7	19.5	1e-6

Tab. 1: Maximum and minimum absorbed power for each term in the power balance, as given by the largest (smallest) Eigenvalue of the respective power correlation matrices.

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