Accelerating MR Elastography with Sparse Sampling and Low-Rank Reconstruction

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INTRODUCTION: Magnetic resonance elastography (MRE) of the brain has emerged as a promising technique for detecting and characterizing neurodegeneration [1] and cerebral tumors [2]. Recent advances in methodology have allowed for greatly improved estimates of mechanical properties, though acquisition of high-quality MRE data remains slow, thus limiting its clinical utility. A typical MRE dataset requires acquisition of full vector displacements in three spatial dimensions, two gradient polarities, and multiple phase offsets. These images share a large amount of information, as they are, in theory, a single magnitude image modulated by harmonic phase patterns. In this work we accelerate MRE through specialized multishot sparse sampling and a low-rank model that captures the strong correlation between images.

THEORY: MRE involves sampling the same object with varying gradient direction, polarity, and displacement

Fig. 1: NRMS accuracy of approximating an experimental MRE data set at different model orders.

phase offsets. Each dataset can be considered a time series of images, $\rho(\mathbf{r},t)$, so highly correlated as to be linearly dependent or low-rank. We can induce low-rankness of $\rho(\mathbf{r},t)$ by modeling it as Lth order partially separable (Eq.1) with ℓ spatial coefficient maps and temporal basis functions $u_\ell(\mathbf{r})$ and $v_\ell(t)$, respectively [3].

$$\rho(\mathbf{r},t) = \sum_{\ell=1}^{L} u_\ell(\mathbf{r})v_\ell(t) \quad (1) \qquad \{\hat{u}_\ell(\mathbf{r})\}_{\ell=1}^{L} = \arg\min_{\{u_\ell(\mathbf{r})\}_{\ell=1}^{L}} \|\mathbf{y} - \Omega(\sum_{\ell=1}^{L} \mathcal{F}\{u_\ell(\mathbf{r})\}v_\ell(t))\|_2^2 \quad (2)$$

where y is the acquired data at undersampled locations indicated by Ω, and $\mathcal{F}$ is the spatial Fourier Transform operator. With appropriate L, the model in Eq. 1 is more robust to experimental imperfections than the conventional harmonic MRE model, yet still strong enough to allow sparse sampling without significant image degradation. The typical MRE dataset consists of 3 directions, 2 polarities, and 8 phase offsets, for a total of 48 images in the time dimension. Thus, the full model order is $L=48$; however, due to the shared information between images, the effective model order of the dataset is much lower. Figure 1 presents the NRMS accuracy of low-rank approximation of an experimental MRE dataset. There is a bend in the curve at $L=6$ (97.4% accuracy), with higher model orders primarily capturing noise instead of signal. If the low-rank temporal basis functions are known (i.e. extracted from a navigator), the low-rank spatial basis functions can be estimated using Eq. 2 and thus the entire dataset can be recovered. This also allows for data to be undersampled in $(\mathbf{k},t)$-space (through undersampling operator Ω in Eq. 2) and reconstructed at low model order without increasing noise level.

METHODS: Human brain MRE data acquired with a multishot spiral imaging sequence [4] was used to investigate the use of low-rank constraints during image reconstruction. The acquisition used six variable-density spiral k-space with 20 slices acquired at a 2x2x2 mm³ isotropic spatial resolution using a Siemens 3T Allegra with a single-channel head-coil and external actuation at 50 Hz. Navigator data were collected from the oversampled portion of each k-space shot, and temporal basis functions were extracted from the singular value decomposition (SVD) of the gridded navigator data. Although the navigators are low-resolution, they share the same temporal basis functions as the high-resolution images of interest. The extracted temporal basis functions were used in two types of reconstructions to demonstrate the performance of rank-constrained MRE. First, all acquired data was used to reconstruct spatial basis functions of varying model order through Eq. 2. In addition to the full-rank reconstruction ($L=48$), we reconstructed datasets using ranks of 22, 15, 10, and 7. Second, we undersampled the data to demonstrate the ability to accelerate acquisitions. Undersampling involved removing acquired shots to achieve 2x, 3x, or 4x acceleration, while requiring each k-space location to be sampled the same number of times over all 48 images, and then reconstructing using a reduced model order. The impact of reduced model order and data undersampling was evaluated through the resulting mechanical property maps. Shear modulus maps were calculated for each dataset using nonlinear inversion (NLI) [5].

RESULTS: Low-rank modeling and sparse sampling can produce displacement fields that still generate the same mechanical property estimates as conventional MRE. Figure 2 presents the z-displacements and real storage moduli of fully

Fig. 2 (left): results of rank-constrained reconstructions with full (L=48) and reduced (L=10) model order. **Fig. 3 (above):** results of undersampling with reduced model order (L=10).

sampled datasets reconstructed with both full rank ($L=48$) and low rank ($L=10$). The displacement fields between the two are nearly identical and give rise to very similar property maps, with a difference in mean properties of less than 1%. Of the model orders we tested, $L=10$ was the lowest that produced highly accurate property maps, so we also used this model order for all undersampled reconstructions. Figure 3 presents displacement fields from datasets reconstructed using varying levels of $(\mathbf{k},t)$-space undersampling, along with the real shear moduli after inversion. The displacement fields show only small differences in wave patterns, and the undersampled reconstructions generally do not exhibit a loss of SNR compared to the full-rank, fully sampled reconstruction in Figure 2. The low-rank constraint allows for less data to be used without decreasing SNR, leading to very similar mechanical property maps at high acceleration factors up to 4x. Differences in mean properties were approximately 1% for 2x acceleration, and approximately 5% for 3x and 4x. These differences are within the expected variations in mean properties [4], and may be addressed by optimizing the sparse sampling scheme.

CONCLUSIONS: We have demonstrated that MRE data can be modeled as a series of partially separable functions, and that using rank-constrained reconstruction of undersampled data results in minimally unchanged property estimates. Given the low model order of typical MRE data, acquisitions can be accelerated by a factor up to 4x without significant loss of SNR or inversion quality. This ultimately can result in high-resolution brain MRE acquisitions on the order of 2-3 minutes, and can be combined with additional parallel imaging undersampling and multishot acquisition schemes for further acceleration.

REFERENCES: [1] I Sack, *et al.*, *Soft Matter*, 2013, 9(24):5672-5680; [2] MC Murphy, *et al.*, *J Neurosurg*, 2013, 118(3):643-648; [3] Z-P Liang, *Proc ISBI*, 2007:988-991; [4] CL Johnson, *et al.*, *NeuroImage*, 2013, 79:145-152; [5] EEW Van Houten, *et al.*,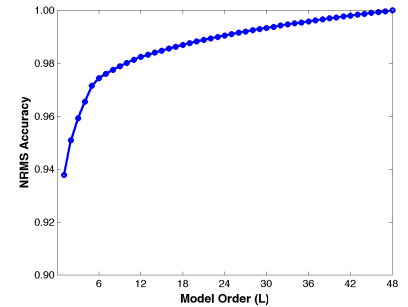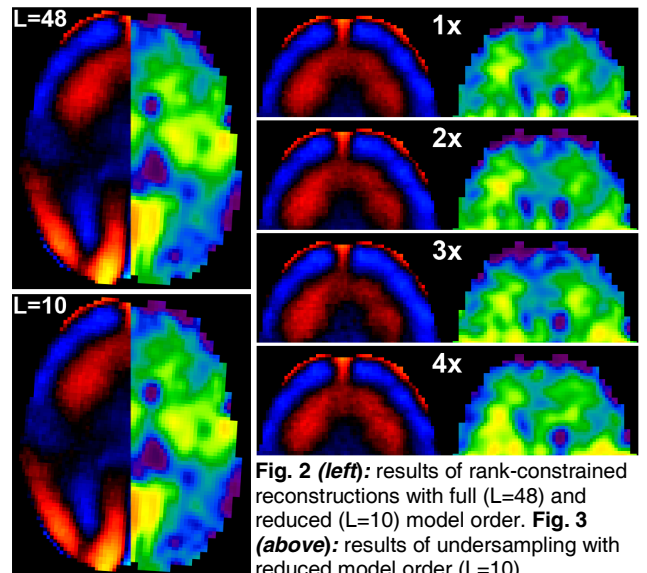 *Med Phys*, 2011, 38(4):1993-2004.