

Obtaining B_1 Distributions by Encoding in B_1 Instead of Image Space

Kalina V Jordanova¹, Dwight G Nishimura¹, and Adam B Kerr¹

¹Electrical Engineering, Stanford University, Stanford, California, United States

PURPOSE: The BEAR method¹ is a recently proposed phase-based B_1 mapping method. An important feature of BEAR is that the phase sensitivity to B_1 can be manipulated with sequence parameters. We describe a new method that calculates the B_1 distribution in a volume from multiple 1D BEAR projections, where each projection has a different B_1 sensitivity. In essence, we are replacing spatial phase encoding with B_1 phase encoding. An estimate of the B_1 distribution in each projected pixel is then calculated using a convex optimization formulation. With this method, an estimate of the B_1 distribution in a volume can be attained faster than by acquiring a 2D B_1 map. Additionally, this method may resolve rapid B_1 variations in space better than with a 2D image. We validate this method through simulations and *in vivo* at 3T.

METHODS: BEAR acquires doubly refocused spin-echo images with phase that is sensitive to B_1 variations using two HSn² adiabatic full-passage pulses. BEAR's phase sensitivity to B_1 can be tuned via the HSn refocusing pulse parameters by varying n , which determines the shape of the pulses, and δ , the ratio of the magnitude of the pulses (Fig. 1).

Multiple successive projection measurements are acquired while varying δ and n_2 to change the B_1 sensitivity. Using the projection measurements and known B_1 sensitivity, an estimate of the B_1 distribution in the volume can be calculated. A given B_1 distribution in a volume can be binned into K bins, each representing a particular B_1 range. If the fraction of spins in the volume with B_1 values in bin k is represented by c_k , then a projection measurement can be written as $p = \sum_k c_k e^{i\phi_k}$, where ϕ_k represents the phase corresponding to $B_{1,k}$ given the known sensitivity, and $k = 1, 2, \dots, K$. If J projections with different sensitivities are acquired, then each projection is $p_j = \sum_k c_k e^{i\phi_{j,k}}$ where $j = 1, 2, \dots, J$. In matrix form: $\mathbf{p} = \Phi\mathbf{c}$, with \mathbf{p} a $J \times 1$ vector containing the measured projections, Φ a $J \times K$ phase matrix, and \mathbf{c} a $K \times 1$ vector representing the unknown B_1 distribution. To solve for \mathbf{c} , a convex optimization formulation³ with additional constraints can be set up as

$$\min_{\mathbf{c}}: \Sigma \text{hub}(\mathbf{p} - \Phi\mathbf{c}) + \lambda_1 \|\mathbf{D}_1\mathbf{c}\|_2 + \lambda_2 \|\mathbf{D}_2\mathbf{c}\|_2, \text{ subject to: } \mathbf{c} \geq \mathbf{0}$$

restricting the solution to be nonnegative real, as desired. This formulation uses the Huber penalty function³ for robustness against outliers. Smoothing constraints have been included as finite difference matrices in the B_1 direction (\mathbf{D}_1), and for adjacent pixels in the 1D projection (\mathbf{D}_2). L-curve corners were used to find $\lambda_{1,2}$ for each pixel, and the mean $\lambda_{1,2}$ were used for optimization. For smoothing, the solution for each pixel used two immediately adjacent pixels.

For both simulated and *in vivo* data, 11 B_1 bins, and 11 sensitivities, uniformly distributed in the range of $\delta = [1, 0.8]$, were used. The number of samples and range in δ are analogous to the number of Y phase encodes and ky excursion for 2D imaging. The B_1 map used for the simulated data was of previously acquired *in vivo* 7T data. To compare actual and measured distributions, the Earth Mover's Distance (EMD)⁴ metric is calculated for each projected pixel, which estimates the work needed to transform one distribution into another. A smaller EMD value indicates greater similarity between distributions.

RESULTS: Fig. 1 shows a range of B_1 sensitivities used in this study (not all sensitivities are shown). The convex optimization B_1 distribution solution closely matches the actual B_1 distribution for simulated data (Fig. 2b-d), with an average EMD of 0.0024 G·(relative counts). *In vivo* (Fig. 2f-h), there is more variation between the measured and actual distributions, with an average EMD of 0.0104 G·(relative counts). Simulated and *in vivo* scanned 2D B_1 maps are shown in Fig. 2a,e for reference. For both datasets, errors in the histograms were concentrated near the actual B_1 distributions of the 2D maps.

DISCUSSION/CONCLUSION: We have demonstrated a new method that estimates the B_1 distribution in a volume without acquiring 2D image data. By encoding the signal data in B_1 , we are able to use multiple measurements to calculate the B_1 distribution. This method was validated in simulation and *in vivo*, with most of the histogram error concentrated along the B_1 distribution. This suggests that errors are most likely due to misplaced counts in consecutive bins, with small errors in B_1 . The convex optimization formulation is a general approach that would allow for the estimation of more B_1 bins than number of encodings, as well as the use of nonuniform B_1 binning. Compared to 2D B_1 mapping methods, this new method may be useful to acquire a faster estimate of the B_1 distribution, or to estimate a distribution where B_1 is varying rapidly in space, such as near a conducting guidewire.

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References: [1] Jordanova *et al.*, Proceedings of ISMRM, Salt Lake City, p. 370, 2013. [2] Tannus *et al.*, NMR in Biomed, 10:423-434, 1997. [3] Michael Grant *et al.*, CVX: MATLAB software. <http://cvxr.com/cvx>, September 2013. [4] Rubner *et al.*, Int. Journal of Comp. Vision, 40:99-121, 2000.

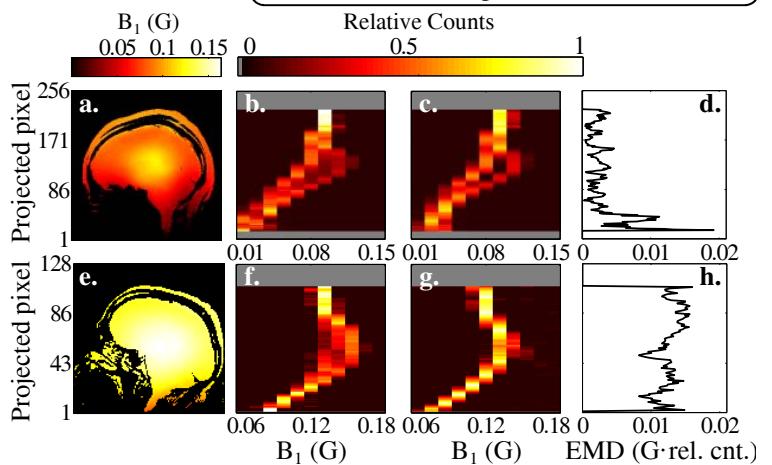
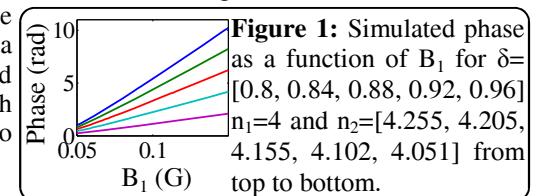


Figure 2: a-d: Simulated and e-h: *in vivo* data and results. a,e: Masked 2D B_1 maps. b,f: Actual and c,g: convex optimization results for the B_1 distribution match visually, with more variation in vivo. d,h: Earth Mover's Distance between (b,f) and (c,g), with mean distances of 0.0024 G·(relative counts) (simulated) and 0.0104 G·(relative counts) (in vivo).