k-SPIRiT: Non-Cartesian SPIRiT Image Reconstruction with Automatic Trajectory Error Compensation

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Target Audience Scientists and engineers interested in image reconstruction.

<u>Purpose</u> Trajectory errors due to eddy current distortions are a significant challenge in non-Cartesian imaging. We propose a SPIRiT-based¹ iterative parallel image reconstruction algorithm for joint estimation of images and k-space trajectory errors: k-SPIRIT. The algorithm is demonstrated in simulations and in vivo experiments.

Theory The algorithm is implicitly based on maximizing data consistency between k-space shots in regions where k-space is oversampled, either by trajectory design or by parallel imaging. The algorithm alternates between nonlinear conjugate gradient (NLCG)-based updates to k-space error weights (while the images are held constant) and SPIRiT image reconstructions using the latest k-space error estimates. An advantage of the method over previous approaches² is that it can be applied to any trajectory, offers greater flexibility in modeling trajectory errors, and both the k-space error updates and image updates

use the same cost function, given by: $\Psi(f,k) = \frac{1}{2} \sum_{c=1}^{N_c} \left\{ \sum_{i=1}^{N_k} \left| y_{ic} - \sum_{j=1}^{N_p} e^{i2\pi([\overline{k_l} + \Delta \overline{k_l}] \cdot \overline{x_j})} f_{jc} \right|^2 \right\} + \lambda \|Gf\|^2$, where f_{jc} is coil c's image, y_{ic} is coil c's data at $\overline{k_l} + \Delta \overline{k_l}$, $\lambda \|Gf\|^2$ the SPIRiT regularization term, and N_c , N_k , N_p , the numbers of coils data samples and image indices respectively. The trajectory

data at $k_t + \Delta k_t$, $\lambda ||Gf||^2$ the SPIRiT regularization term, and N_c , N_k , N_p , the numbers of coils, data samples, and image indices, respectively. The trajectory error $\Delta \vec{k}$ is constrained by modeling it as a sum of N_b weighted basis functions, for each shot $e \in \Lambda k$, $f(t) = \sum_{i=1}^{N_b} n_i f(t) w_i$, where w_i , are the weights to be

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Figure 1. (a) Center of k-space for nominal and randomly shifted trajectories for simulated phantom (b) Corrected trajectory overlaid on shifted trajectory (c) SPIRIT reconstructions of phantom on uncorrected (left) and corrected (right) trajectories.

for each shot, e.g.: $\Delta k_x(t) = \sum_{b=1}^{N_b} p_{xb}(t) w_{xb}$, where w_{xb} are the weights to be estimated for each shot x. Selection of appropriate basis functions is critical to the method's performance; for radial imaging, constant functions allowing each projection to translate in k_x/k_y are a natural choice.² For arbitrary trajectories, basis functions can be generated using eddy current models with multiple time constants, and can be compressed by SVD.

Methods The algorithm was tested on simulated radial phantom data (64x64 matrix, 8 coils, 25 projections), with each of the lines in a typical radial trajectory receiving its own random translation in k_x/k_y (i.e. constant error basis functions) up to a maximum of 1/FOV. Figure 1a shows the nominal radial trajectory and the shifted lines actually sampled. We applied k-SPIRIT to correct for these shifts using 7 iterations, each consisting of 5 NLCG k-space error updates followed by a 25-iteration SPIRiT image update. We also collected in-vivo brain data at 7T (Philips Achieva, Philips Healthcare, Cleveland, OH) with a ramp-sampled, 1.15 ms readout radial center-out trajectory (128x128 matrix, 32 coils, 192 center-out projections, 24 cm FOV). Six error basis functions were generated from an SVD-compressed set of decaying exponential functions with varying time constants convolved with the derivative of the nominal gradient.³ To reconstruct a 'true' image, we measured the same gradient waveforms on the scanner using a modified Duyn's method.⁴

Results Figure 1b shows the corrected simulation trajectory, which overlaps the true trajectory. Figure 1c shows SPIRiT reconstructions of the phantom on the nominal and corrected trajectories. Figure 2 shows SPIRiT reconstructions of the in-vivo brain data on the measured, nominal, and corrected trajectories. RMS trajectory errors were 0.043 cycles/cm and 0.017 cycles/cm for the nominal and corrected trajectories, respectively.

<u>Discussion and Conclusion</u> The proposed algorithm corrected for the shifts in the simulated trajectory, even with an acceleration factor of about 4, and removed the streaking artifacts present in the phantom image as seen in Figure 1c. The in-vivo image reconstructed using the corrected trajectory parallels that reconstructed using the measured trajectory, and substantially improved on

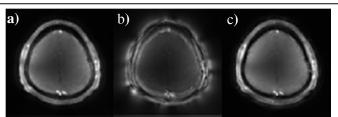


Figure 2. SPIRiT reconstructions of 7T in-vivo brain data along (a) measured, (b) nominal, and (c) corrected trajectories.

the nominal trajectory reconstruction. Note that the method can be applied to multicoil datasets and is aided by SPIRiT, but it does not strictly require multiple coils. Furthermore it may apply to other non-Cartesian trajectories, such as spirals.

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References 1. M Lustig, JM Pauly, MRM 64: 457–71, 2010. 2. T Wech et al, ISMRM 21:0129, 2013. 3. JJ Van Vaals, AH Bergman, JMR 90: 52–70, 1990. 4. P Gurney et al, ISMRM 13:866, 2005.