Non-Iterative Bayesian Reconstruction Algorithm for Undersampled MRI Data

Gengsheng Lawrence Zeng[1] and Edward V. R. Dibella[1]
[1]*Radiology, University of Utah, Salt Lake City, UT, United States*

Introduction

The Fourier transform based image reconstruction is standard practice in MRI. However, when constraints need to be incorporated in the image reconstruction, iterative algorithms are utilized. One drawback of iterative reconstruction methods is the relatively longer computation time. Another issue is that the iterative algorithm must stop with a finite number of iterations and may not reach the optimal solution. This paper proposes a closed-form solution for the MRI reconstruction problem with Bayesian constraints. This paper considers a problem, where the *k*-space is undersampled at each time frame, but some data are available at other the *k*-space locations at other time frames. Data at other time frames can assist the image reconstruction at the current time frame. This data assisting image reconstruction has been used in many contexts such as with iterative spatio-temporal constrained reconstruction (STCR) algorithms for cardiac perfusion [1]. The current paper applies a newly developed closed-form Fourier-transform-based algorithm [2] to our undersampled MRI example, in which the *k*-space is radially sampled.

Methods

An objective function, F = (fidelity term) + β (Bayesian term), is first set up as a sum of a quadratic data fidelity term using the current time-frame data $P1$ (referred to as the primary data) and a quadratic Bayesian term using the combination of immediately-before, current, and immediately-after data $P2$ (referred to as the secondary data), where β controls the influence of the secondary data. The calculus of variation method is applied to the objective function and a closed-form optimal solution is then obtained. This optimal solution is expressed in the Fourier domain and can be implemented as follows. (i) Prepare two sets of projection data $P1$ and $P2$. The primary data $P1$ at each time frame consist of current 24 *k*-space radial lines. The secondary data $P2$ are the time-averaged most recent 96 *k*-space radial lines. (ii) Apply the conventional ramp filter $|\omega|$ to the secondary projection set $P2$ for each line, obtaining Q. (iii) Form the new projection data set: $P1 + \beta Q$. (iv) Apply the newly derived "modified ramp filter" $|\omega|/[1+\beta\cdot|\omega|]$ to the combined data set. (v) Perform conventional backprojection that maps the "projections" into an image.

Results

The data were acquired with a Siemens 3T Trio scanner, using phased array of coils, of which one coil was chosen to demonstrate the method. The acquisition matrix size for an image frame was 256 x 24, and 60 sequential images were obtained at 60 different times. At each time frame, the *k*-space is sampled with 24 uniformly spaced radial lines over an angular range of 180º; however, the 24-line sampling patterns of the adjacent time frames are offset by 180º/96. Two sets of reconstructions are shown in Fig. 1. The conventional filtered backprojection method (shown in row 1) uses only the current time frame data. The proposed closed-form Bayesian reconstruction (shown in row 2) uses both primary and secondary data. There are some significant image intensity changes and movements for different time frames. The first two row images are displayed from the minimum to maximum of each individual image. The 3rd row is the difference of the first two rows, and purpose of the 3rd is to show that the myocardium concentration in the 2nd row follows faithfully of that in the 1st row. The images in the 3rd row are displayed using a global minimum and maximum gray scale.

Conclusions

This paper introduces a new Fourier transform based Bayesian algorithm, which is non-iterative. The advantages of this new algorithm over the iterative algorithms are its fast computation time and the fact that it can actually reach the optimal solution. We observe that when the primary and secondary projection data sets are to be combined. These two data sets cannot be combined by a simple weighted summation. The secondary data set needs to be pre-filtered by a ramp filter before the weighted summation can be performed, in which the DC component is removed and the low frequency components are significantly suppressed.

This approach is different from the HYPR-type methods [3], which were derived in an *ad hoc* manner and do not work well when the object is in constant motion. However, our method is analytically derived to be the optimal solution of a minimization problem and is able to track the object motion.

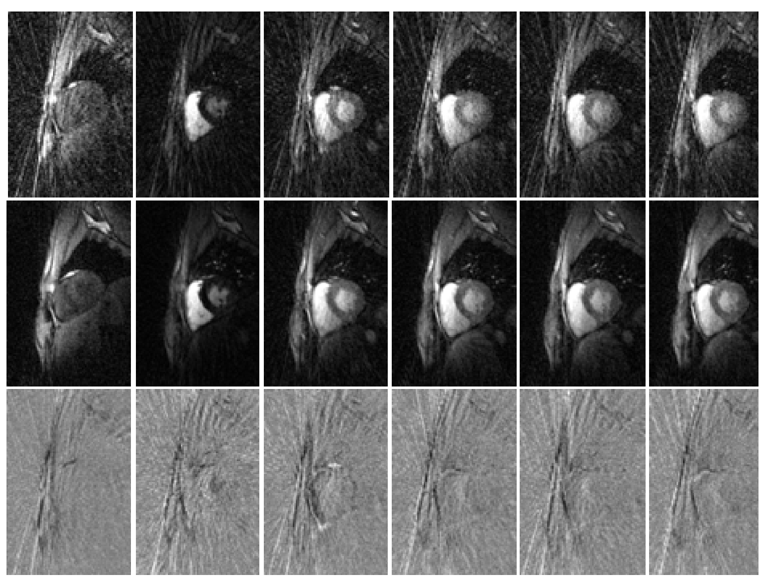

Fig. 1. Comparison of conventional Fourier reconstruction (1st row) and proposed Fourier-Bayesian reconstruction (2nd row), using undersampled dynamic MRI data. The 3rd row shows the difference images between the 1st and 2nd rows. Six time frames are shown from left to right.

References

[1] G. Adluru, *et al*, "Acquisition and reconstruction of undersampled radial data for myocardial perfusion MRI," *J. Magn. Reson. Imaging*, **29,** 466-473 (2009).
[2] G. L. Zeng, "A filtered backprojection MAP algorithm with non-uniform sampling and noise modeling," *Med. Phys.*, **39**, 2170-2178 (2012).
[3] C. A. Mistretta, *et al*, "Highly constrained backprojection for time-resolved MRI," *Magn. Reson. Med.*, **55**, 30-40 (2006).