Magnetic resonance elastography for the measurement of the bulk modulus in compressible materials.

Sebastian Hirsch[1], Frauke Beyer[1], Sebastian Papazoglou[1], Jing Guo[1], Juergen Braun[2], and Ingolf Sack[1]
[1]*Department of Radiology, Charité - Universitätsmedizin Berlin, Berlin, Berlin, Germany,* [2]*Institute of Medical Informatics, Charité - Universitätsmedizin Berlin, Berlin, Berlin, Germany*

Target audience: Scientists and clinicians with an interest in MR Elastography (MRE) or in the measurement of tissue pressure.
Purpose: To assess and demonstrate the capability of MRE for measuring the bulk modulus of compressible materials.
Methods: Compressible gel phantoms of various densities ρ were produced by mixing hot agarose gel, ultrasound gel, $NaHCO_3$ and citric acid in order to form CO_2-filled cavities after solidification. A torsional rheometer (Anton Paar, Physica MCR 301) operated in the axial (compression) mode was used on 23 phantom slices to measure the bulk modulus K as a function of the phantom density ρ. Then, MRE with 50 Hz harmonic excitation frequency was applied to three large cylindrical phantoms (∅ 6 cm, length 20 cm) with densities $\rho = 0.89$, 0.93 and 0.95 g/cm³. The full 3D-wave field **u** was acquired at 6 different vibration amplitudes. The wave equation for the compression component of the displacement field, $M = -\rho\omega^2 \frac{\nabla\bullet\mathbf{u}}{\nabla\bullet\nabla(\nabla\bullet\mathbf{u})}$ with angular excitation frequency ω, was solved for the P-wave modulus $M=K+4/3\mu$ (μ: shear modulus) by averaging the magnitudes of the nominator and denominator over a region of interest. Results from the rheometer experiments were compared to MRE by employing an effective medium model with variable ρ. Furthermore, the effect of noise to the MRE data was investigated by numerical simulations. Therefore, the longitudinal displacement field in a homogeneous medium of M = 1 MPa was simulated with various levels of Gaussian noise. The generic data were analyzed for M employing symmetric-difference gradient operators and a multi-dimensional averaged gradient scheme according to Anderssen and Hegland[1].
Results: Fig. 1 shows K measured in 23 compressible phantoms by the rheometer. Each symbol represents a phantom with distinct ρ. The blue line corresponds to a model suggested by Wood[2], $K = p\cdot K_{gel}/((1-\frac{\rho}{\rho_0})K_{gel}+\frac{\rho}{\rho_0}p)$ with the compression modulus of the pure gel K_{gel}, cavity gas pressure p and ρ_0 = 1 g/cm^3, fitted to the experimental data by K_{gel}=6.5 MPa and p=0.03 MPa. Fig. 2 presents data from the MRE experiments. Each cross corresponds to an MRE experiment applied to one of the three phantoms (1-softest [blue], 3-stiffest [green]) at a given vibration amplitude (increasing from bottom left to top right). M-values were deduced from the slopes of the solid fit lines and tabulated in Tab.1. The systematic underestimation of M in the presence of noise is illustrated in Fig. 3.

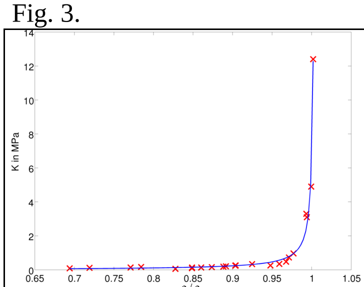

Fig. 1: Results from the rheometer experiment.

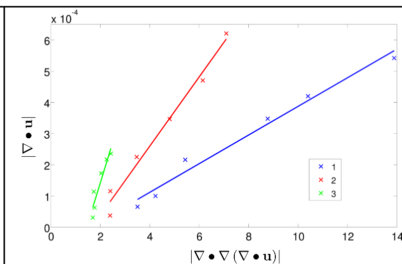

Fig. 2: Results from the MRE experiment.

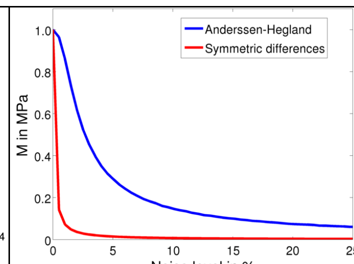

Fig. 3: Results from the noise simulation.

Phantom	Density (g/cm^3)	*M* (kPa)
#1	0.89	4.5
#2	0.93	10.9
#3	0.95	25.4

Tab. 1: Reconstructed P-wave moduli (MRE)

Discussion: The proposed effective-medium compression model is corroborated by the experimental rheometer data, although both K_{gel} and p were smaller than expected. MRE was capable to reproduce the relative order of the compression modulis of the phantoms (#3 > #2 > #1). However, the absolute values of M measured by MRE are 1-2 orders of magnitude smaller than the values predicted by the rheometer experiments. The simulation provides insight into the origin of this bias: A rapid decrement of the reconstructed M-modulus is seen in the presence of noise, which can partially be alleviated by the more robust Anderssen-Hegland gradient scheme. Consequently, future studies of compression-sensitive MRE should account for the potential underestimation of M due to noise.
Conclusion: An effective medium model was introduced to MRE for measuring the bulk modulus of compressible materials. The model was validated by rheometer experiments and simulations. Compression-sensitive MRE is capable of detecting pressure- and density-related mechanical parameters and therewith provides a means of investigating the poroelastic properties of in vivo tissue. In future studies the high sensitivity of compression MRE to noise must be accounted for by considering the relative change of tissue compressibility under the influence of physiological processes or diseases.

References: 1. Anderssen RS, Hegland M. For Numerical Differentiation, Dimensionality Can be a Blessing. *Math Comput*. 1999; 68: 1121-1141 2. Wood AB. *A textbook of sound*. London: G. Bell and Sons ltd; 1930