Noise Robust Inverse Laplacian Operator Based Reconstruction of Global Elastic Parameters in Magnetic Resonance Elastography.

Sebastian Papazoglou[1,2], Heiko Tzschaetzsch[1], Juergen Braun[3], and Ingolf Sack[1]
[1]*Radiology, Charité University Medicine, Berlin, Berlin, Germany,* [2]*NeuroCure Clinical Research Center, Charité University Medicine, Berlin, Berlin, Germany,*
[3]*Medical Informatics, Charité University Medicine, Berlin, Berlin, Germany*

Background: Magnetic resonance elastography (MRE) is an image-based modality for the non-invasive assessment of viscoelastic properties of in vivo soft tissues [1]. Although MRE has been extremely useful for the detection of hepatic fibrosis, it still suffers from an uncertainty of elasticity parameters in presence of noise, e.g. in MRE on human lung [2]. Main reason is the regularly used direct inversion of the wave equation (DI) [3], which invokes noise-enhancing second derivatives of the field in the denominator leading to strongly underestimated elastic parameters [4]. As a result, MRE provides consistent viscoelastic parameters only by considering spatially averaged values related to the constitution of the whole organ. The aim of the present study was to develop a global viscoelastic parameter reconstruction method which i) avoids noisy gradient data in the denominator and ii) which inherently averages the complex shear modulus μ during the inversion procedure. The method is based on the formulation of the wave equation by an inverse Laplace operator which directly provides the desired modulus μ.

Theory: As a generic model for time harmonic shear waves, the well-known Helmholtz equation in Cartesian coordinates was employed. In an isotropic and homogenous elastic medium it reads for any Cartesian displacement component

$$\mu\Delta u(x,y,\omega) = -\rho\omega^2 u(x,y,\omega)\,, \qquad (1)$$

where μ is the global shear modulus, Δ is the Laplace operator, $u=u(x,y,\omega)$ is the complex displacement component, ρ is the mass density of the tissue under investigation and ω is the angular frequency which is related to the external MRE drive frequency f by $\omega=2\pi f$. Eq. (1) can be rewritten as

$$\mu\mathbf{Lu} = -\rho\omega^2\mathbf{u}\,, \qquad (2)$$

where **L** is the discrete matrix version of Δ and **u** is a column vector. Equation (2) is an overdetermined system of linear equations for μ. Eq. (2) may now be solved in two very distinct ways. First, we formulate the direct inversion (DI) in a weighted least-squares approach, i.e.:

$$\mu = -\rho\omega^2 \frac{(\mathbf{Lu})^T \cdot \mathbf{u}}{(\mathbf{Lu})^T \cdot (\mathbf{Lu})}\,,\ (\mathrm{DI}) \qquad (3)$$

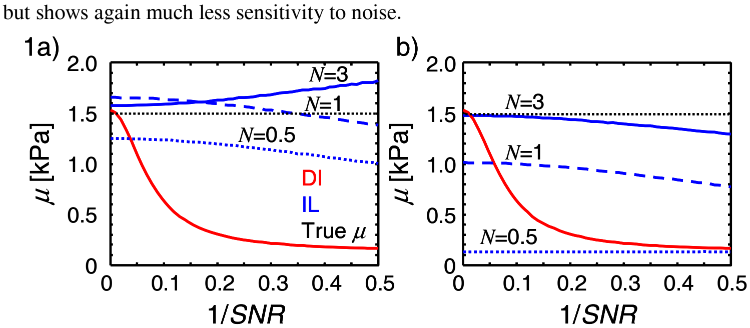
where T denotes transpose and complex conjugate. On the other hand eq. (2) allows to apply the matrix inverse of the Laplace operator first and then to solve in analogy to eq. (3):

$$\mu = -\rho\omega^2 \mathbf{u}_0^T \mathbf{L}^* \mathbf{u}_0\,,\ (\mathrm{IL}) \qquad (4)$$

where $\mathbf{u}_0=\mathbf{u}/|\mathbf{u}|$ and $\mathbf{L}^*$ is an appropriate matrix inverse, depending on whether **L** incorporates boundary pixel ($\mathbf{L}\equiv\mathbf{L}_1$ is a square matrix, thus $\mathbf{L}_1^*\equiv\mathbf{L}_1^{-1}$) or not ($\mathbf{L}=\mathbf{L}_2$ is a non-square matrix and $\mathbf{L}_2^*$ is the Moore-Penrose pseudoinverse). In the following eq. (4) will be referred to as inverse Laplacian (IL) method.

Methods: The IL method was compared with DI using simulated complex 1D and 2D scalar waves. Plane waves were simulated at f=50 Hz vibration frequency, with μ= 1.5 kPa and a pixel size of 1 mm. To assess the sensitivity with respect to noise, normally distributed noise at values for 1/*SNR* between zero (no noise) and 0.5 (very noisy data) was added to both real and imaginary part of the wave independently. A total of 1000 noise configurations was generated to estimate the mean of the reconstructed μ. The number N of wave lengths in propagation direction was varied between 0.5, 1, 2 and 3 to investigate the influence of the size of the data set utilized in inversion. For 2D the width of the field of view (FOV) used in inversion was held constant at 20 pixels.

Results: Fig.1a) shows the reconstructed μ in 1D from DI and IL using $\mathbf{L}_1$. Clearly, DI is affected most by noise. Above 1/*SNR*=0.05 the elasticity becomes strongly underestimated. In contrast the IL method yields reasonably well μ values in case of 0.5 to 1 wavelength contained in the FOV over the entire range of 1/*SNR* studied. However, at increasing number of wavelengths in the FOV it overestimates elasticity particularly at large 1/*SNR*. This is not seen in **fig**. 1b), where the same scenario is shown using the Laplacian $\mathbf{L}_2$. At increasing N the recovered μ approaches the value used in the simulations showing little influence due to noise at N=3 at large 1/*SNR*. **Fig.** 2) shows exemplary five 2D FOVs at varying 1/*SNR* at fixed N=3 evaluated in the computational experiments. The results obtained with DI and IL using $\mathbf{L}_1$ are summarized in **fig.** 3). Again, it is observed that DI strongly underestimates μ. Similarly, also the IL method underestimates μ but shows again much less sensitivity to noise.



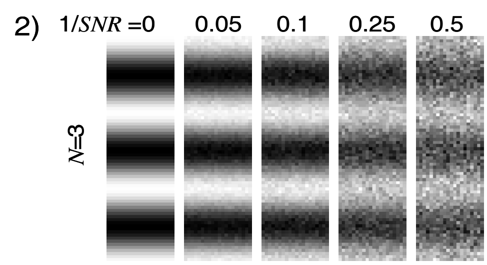


Fig. 1: Reconstructed μ in 1D by DI and IL for different N: a) using $\mathbf{L}_1$ (i.e. with boundary pixel), b) using $\mathbf{L}_2$ (i.e. omitting boundary pixel). True value used in the simulations was μ=1.5 kPa.

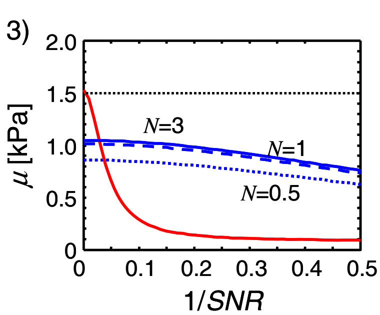


Fig. 2: Sample 2D FOV used in inversion at different 1/SNR.

Fig. 3: Reconstructed μ in 2D obtained by DI and IL using $\mathbf{L}_1$ (with boundary pixel). True value used in the simulations was μ=1.5 kPa.

Discussion and Conclusions: The IL method was introduced to circumvent the noise amplifying effect of derivatives and the numerically instable division by oscillating wave fields. As observed by comparison with results due to DI shown in **fig.** 1) and 3) the IL shows superior performance over DI especially at large 1/*SNR*, where the strong underestimation renders DI reconstruction ineligible. However, as a consequence of matrix inversion, fixed boundary conditions implied by $\mathbf{L}_1$ spread over the entire domain biasing recovered μ values as seen in **fig**. 1a). Using the Laplace operator $\mathbf{L}_2$ with non-fixed boundary conditions mitigates this bias and leads to correct values of μ at increasing N in 1D. In 2D (**fig**. 3) the insensitivity of IL is retained but the correct value for μ is not fully recovered suggesting pronounced influence of boundary pixel.

This study demonstrates that the IL method is a very promising approach due to its inherently very low sensitivity with respect to noise. Further investigations are required on the influence of the pixel per wavelength resolution on the elastic parameter reconstruction and the dependence of reconstructed elasticity on the width of the employed FOV.

References: [1] Muthupillai, M. et. al., Magn. Reson. Med., **36**(2), 266-276, 1996, [2] Hirsch, S. et. al., Magn. Reson. Med., doi:10.1002/mrm.24294, 2012, [3] Oliphant, T.E. et. al., Magn. Reson. Med., 45(2), 299-310, 2001, [4] McGee, K.P. et. al., Proc. Intl. Soc. Mag. Reson. Med. **19**, 2011