MR Physics for Physicists: Electromagnetic Fields in MRI: From Theory to Practice

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Title: *Static Magnetic Field: From magnetic susceptibility to Magnetic Field (in)Homogeneity and back*

Summary

This lecture will address the concept of magnetic susceptibility and its interaction with strong static magnetic fields - the demagnetizing field. In the context of magnetic resonance imaging the demagnetizing field, depending on its strength and MR imaging parameters, can cause: image distortions, signal attenuation through intravoxel dephasing or variation of the voxel mean magnetic field that can be observed in the phase of gradient echo images.

To benefit from this lecture it is important to be familiar with image formation and encoding using gradients as well as the basic Bloch Equation describing the signal evolution under a static magnetic field.

The lecture will focus on the information observed in gradient echo phase images, and on the methods developed over recent years to (having measured local variations of the static magnetic field) recover the information from the underlying magnetic susceptibility. The lecture will also cover some of the lessons learned regarding the magnetic properties of tissues in the human brain using susceptibility mapping and some of the limitations of these techniques.

At the end of the course the concept of demagnetizing fields should be clear and the potentials and pitfalls of quantitative susceptibility mapping should be understood.

Magnetic susceptibility

Materials can be, in terms of their susceptibility, divided into three classes: Diamagnetic (small negative χ); Paramagnetic (small positive χ) and Ferromagnetic (positive χ). The susceptibility present in brain tissues is generally diamagnetic - as most molecules and compounds present in the brain do not have unpaired electrons. Therefore, the field perturbation (ΔB_0) generated by these diamagnetic compounds is very small compared to the external magnetic field. As a result it is possible to describe its effect as simply a convolution of the field generated by a magnetic dipole with that of the susceptibility distribution and to focus on its projection parallel to the externally applied magnetic field. This dipolar demagnetizing field can be analytically described in Fourier space as:

$$\Delta\tilde{B}_0(\vec{k}) = -B_0\tilde{\chi}(\vec{k})\left(\frac{k_z^2}{k^2}-\frac{1}{3}\right) \qquad \text{Eq. 1}$$

Where k represents the coordinates in k-space, k_z the coordinate along the direction parallel to the main magnetic field (B_0) and $\tilde{\chi}(\vec{k})$ is the Fourier transform of $\chi(\vec{r})$ [1, 2].

Similar – or equivalent - descriptions of the magnetic field perturbation of an arbitrary (or not) shape of magnetic susceptibility have been extensively used to understand image distortions in

MRI [3] and T2* decay [4] (due to, for example, varying oxygenation levels in the venous system responsible by the BOLD effect).

Phase Imaging

More recently, it has been noted that the phase of gradient echo images were not only useful to obtain fieldmaps characterizing the field inhomogeneitiees arising from the susceptibility differences between air and tissue, but showed interesting anatomical detail with clear depiction of the vasculature and of contrast between and within grey and white matter structures [5, 6].

Using phase imaging, various groups have tried to analyze the compounds able to explain the observed contrast in the brain. The current consensus being: (1) the contrast observed between veins and tissue is modulated by deoxyhemoglobin [7]; (2) the grey/white matter contrast is mainly modulated by myelin concentration [8-10]; (3) deep grey matter contrast and some cortical layers are mainly modulated by the presence of different forms of iron [11-13];

Susceptibility mapping

The observed phase contrast $\phi(\vec{r}) = TE \cdot \gamma \cdot \Delta B_0(\vec{r})$ and the measured field are not fully quantitative measures able to directly characterize different tissues due to their non-locality. The linear relationship described by Eq. 1 can be used to compute the susceptibility distribution. Inverting this linear system is not trivial, as the inversion of the dipole kernel is not defined when $k^2 - 3k_z^2 = 0$. This means that the inverse problem is ill-posed and there are an infinite number of susceptibility distributions that can give rise to the same relative frequency shift. One way of dealing with this problem [1, 14] is to measure the object in various directions with respect to the main magnetic field, generating an over-determined problem that can be simply solved by least squares minimization. Although accurate, this technique needs three separate measurements in order to compute susceptibility maps. Such an approach is not very practical as rotations of ~20 degrees are not practical to perform in a conventional MR scanner, and makes the post-processing analysis more complex. Alternatively, many advanced regularization methods have been introduced in order to compute the susceptibility map associated with a fieldmap measured using one single head position [15-20].

While susceptibility in liquids and emulsions is simply a scalar term, in structured materials it is better described by a tensor. Such observations have now been confirmed in white matter bundles, both by elegantly designed ex-vivo experiments [21] and by directly measuring this tensor through the acquisition of very large number of head positions [22].

Pitfalls

Despite the rich information observable in the reconstructed susceptibility maps, their results should be interpreted with some caution. One of the main complications is that susceptibility maps, although quantitative, should always be considered relative (either to CSF to some other reference) due to the fact that Eq. 1 is not defined for k=0. Another more fundamental source of skepticism arises from the fact that Eq. 1 is only valid for homogeneous distributions where the Lorentz sphere approximation holds. Simple geometric arguments have been used to demonstrate that this does not have to be the case for structured samples with non-punctuate sources of susceptibility [23] such as white matter. Furthermore, the existence of different water compartments with different water concentration and different relaxation parameters implies

that the observed frequency shift in a given voxel is not the simple average field in the voxel, but a weighted average. This weighted average can be either defined by the different water concentration of each compartment or their different relaxation parameters. Hence, the measured fieldmaps could then be considered to be sequence parameter dependent and have been shown to vary with echo times [24]. Such violations of Eq. 1, along with other suggested contrast mechanisms[25, 26], form the basis of some of the caution needed when interpreting susceptibility maps.

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