Maxwell Equations and EM Modeling

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Target audience. Built on a basic physical understanding of classical electrodynamics as instructed in graduate physics courses, in this lecture electromagnetic (EM) models represented by EM field, current, and charge distributions in space obtained from numerical computations are explained and how they arise from analytical forms of Maxwell's equations. The MRI physicist shall gain a principal overview on the mathematical background of how to translate the analytical form into numerical algorithms and how boundary conditions are taken into account.

Objectives. In textbooks Maxwell's equations are often written in their elegant form as first-order differential equations. Solving them to attain the electric and magnetic fields requires taking into account specific boundary conditions imposed by the particular system under consideration. In the lecture it will be explained how Maxwell's equations can be solved numerically and, in the case of complex boundaries and material distributions, why it is more straightforward to apply integral forms of Maxwell's equations, because boundary conditions may lead to discontinuous and non-differentiable functions, which can be dealt with in a sound manner by advanced techniques of integral calculus [1]. Furthermore, setting up discrete EM models to obtain electric and magnetic fields appears less complicated when using integral forms as a starting point. In these discrete models space is partitioned into small cells which form a mesh and the EM model is computed on this mesh.

Methods and Examples. The procedures involved are exemplified by elucidating one particular class of models based on algorithms of the finite integration technique (FIT) introduced by Weiland [2], appropriate for numerically solving Maxwell's equations. The role of meshing 3D space will be explained. Two particular model examples are discussed: (a) an EM model for the propagation of a microwave beam inside an NMR probe that allows dynamic nuclear polarization (DNP) experiments – here the wavelength (millimeter or sub-millimeter) of the high-frequency field distribution is short compared to or of the same order of magnitude as the sample or the probe, and (b) an EM model of a single loop used as an MRI surface coil for the human head, where the wavelength (several meters) is much larger than the spatial extension of the sample.

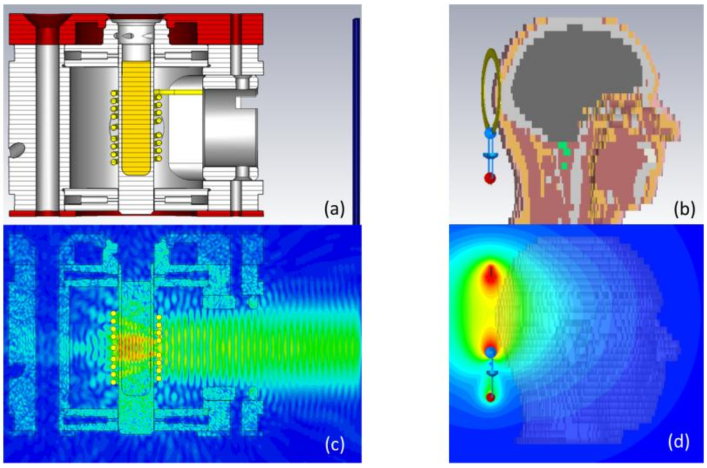

Figure 1: Geometrical cross sections through example structures (a) and (b) and numerically computed magnetic field distributions in magnitude representation (c) and (d) at frequencies of interest (263 GHz and 114 MHz, respectively).

Discussion. Although nowadays many sophisticated, efficient, and far-developed numerical codes exist to solve EM problems, it is essential to validate results obtained by numerical computation with indicators obtained through experiments. Only the comparison of simulation data with typical experimental situations wherever possible provides the certainty and safety desired for EM modeling and allows the application of models extended to circumstances which are not directly accessible by measurement.

References

[1] A. Bossavit, Computational Electromagnetism, Academic Press, San Diego, 1998.

[2] T. Weiland, Particle Accelerators **15**, 1984 and **17**, 1985. T. Weiland, IEEE NS **32**, 2738, 1985. T. Weiland, Phys. Bl. **42**, 191, 1986. M. Clemens, T. Weiland, Progr. Electromagn. Res., PIER **32**, 65, 2001.